DOCUMENT RESUME

ED 061 384 24 UD 012 228

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TITLE Optimal Allocation of Resources in Urban Education: An Econometric Approach. Final Report.
INSTITUTION Northwestern Univ., Evanston, Ill.
SPONS AGENCY Office of Education (DHEW), Washington, D.C. Bureau of Research.
BUREAU NO BR-0-0334
PUB DATE Aug 71
GRANT OEG-5-70-0026(508)
NOTE 89p.

EDRS PRICE MF-$0.65 HC-$3.29
DESCRIPTORS Academic Achievement; Academic Aptitude; Academic Performance; Educational Accountability; *Elementary Grades; Grades (Scholastic); *High Schools; Language Development; Learning Difficulties; Low Achievers; Motivation Techniques; Performance Contracts; Reading Achievement; *Student Motivation; Urban Areas; *Urban Education; *Verbal Ability; Verbal Development; Verbal Learning; Verbal Operant Conditioning

ABSTRACT

This study identifies the policy variables that are effective in increasing student verbal achievement in urban grammar schools and high schools, and estimates the impact of these variables upon verbal achievement, expected years of education completed by a typical student, and expected lifetime earnings of a typical student. A theoretical model of student motivation is developed and used as a guide in the specification of an empirical model of student achievement. The empirical model is estimated using the data gathered for the following: (1) research methodology; (2) summary of results, conclusions, and recommendations; and (3) analytic and empirical background for the conclusions and recommendations. Included in the third section are geometric and mathematical treatments of the data, replete with numerous tables, figures, and charts. (Author/SB)

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Final Report

Project No. 0-0334

Grant No. OEG-5-70-0026 (508)

OPTIMAL ALLOCATION OF RESOURCES IN URBAN EDUCATION: AN ECONOMETRIC APPROACH

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August 1971

The research reported herein was performed pursuant to a grant with the Office of Education, U.S. Department of Health, Education, and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their professional judgment in the conduct of the project. Points of view or opinions stated do not, therefore, necessarily represent official Office of Education position or policy

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UD 012 228

OPTIMAL ALLOCATION OF RESOURCES IN URBAN EDUCATION: AN ECONOMETRIC APPROACH

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PART I INTRODUCTORY SECTION

SUMMARY

This study identifies the policy variables that are effective in increasing student verbal achievement in urban grammar schools and high schools. A theoretical model of student motivation is developed and used as a guide in the specification of an empirical model of student achievement. The empirical model is estimated using the data gathered for the Coleman report on Equality of Educational Opportunity. The empirical model is used to quantify the rates of return associated with each policy variable

INTRODUCTION

The purpose of this study is to identify an economically and educationally efficient pattern of resource allocation in urban education. To this end, the study identifies the policy variables that are effective in increasing student verbal achievement in urban grammar schools and high schools and estimates the impact of these variables upon verbal achievement, upon expected years of education completed by a typical student and upon expected lifetime earnings of a typical student. Previous studies of resource allocation in education have had some difficulty in distinguishing effects which are specific to particular, school controlled, policy variables because of the large amount of intercorrelation among these variables and between these variables and variables describing the students' socio-economic status and non-school environment.

METHODS

The present paper develops a theoretical model of student motivation that is used as a guide in the specification of an empirical econometric model of student achievement. This empirical model describes the educational process in grammar schools and in high schools as a chain of causal relationships, thereby greatly reducing the imprecision due to intercorrelation among the student background and the school variables. The empirical model is used to estimate, by means of econometric techniques, a simultaneous equation econometric model. The parameters of the estimated model are used to derive estimates of the impact of the school controlled policy variables upon various indicators of school output: student verbal achievement, expected years of school completed, and expected student earnings. Further validation of the structure of the empirical model is provided in Appendix B.

The Model of Student Achievement

The theory of student motivation developed here is essentially an outgrowth of the theory of consumer choice. The latter theory describes how the consumer's dollar budget is allocated among various goods. The present theory describes how the student chooses to allocate his classroom time between academic and non-academic pursuits. In the theory of consumer choice, the consumer is assumed to purchase the satisfaction embodied in the consumption of a bundle of goods and services with the money he spends. Here the student is viewed as purchasing a stream of present and future rewards with the time at his disposal.

It is assumed that the student divides his classroom time among time spent on academic pursuits and time spent on non-academic pursuits with a view to maximizing the psychological rewards (utilities) resulting from this allocation. In the analysis that follows, the proportion of classroom time spent on academic pursuits is viewed as allocated to the purchase of a composite good comprising the rewards (teacher acceptance, expected future earnings, etc.) which can be purchased with classroom time devoted to academic work. Similarly, the proportion of time spent on non-academic pursuits is viewed as devoted to the purchase of another composite good consisting of the rewards (peer acceptance, leisure) that tend to be acquired by expenditure of classroom time on non-academic endeavors. It is shown how various school and socio-economic factors affect the students' evaluation of the two composite goods which can be purchased with academic and non-academic usage of classroom time (i.e., their utility functions) and how these variables influence the ability of students to transform classroom time into peer group and into teacher acceptance (i.e., their opportunity sets). The result is a theory which describes the way in which student study habits are affected by various school and home characteristics. That theory is then used to specify how these variables should enter a descriptive empirical model that predicts the level of student achievement.

The Econometric Models

The model thus arrived at is estimated econometrically, using simultaneous equation techniques. Separate equations are estimated for a grammar school and for a high school model.

The general flow of causation in the estimated models can be summarized as follows: In the first equation student verbal achievement is determined by student motivation and several other school and background variables. In the second equation student motivation is determined by prior verbal ability and the extent of disciplinary problems in high schools and by parental interest, effective class

size and several other variables in grammar schools. The third equation for grammar schools determines effective class size by actual class size, the quality of instructional equipment and by the extent of disciplinary problems in the school. The subsequent equations determine the extent of classroom disciplinary problems in the high schools and grammar schools by actual class size, parental interest in education and several other school and background variables. Finally, parental interest in education is determined by family background characteristics. The flow of causation in the model therefore runs from various school and background variables through the extent of disciplinary problems and the opportunity for personal contact between teacher and student, which interact to determine student motivation and therefore student achievement.

The endogenous variables in the model are student verbal ability in the sixth and twelfth grades, student motivation, a proxy for the number of positively reinforcing contacts that can be made by the teacher[1], student disciplinary problems, and parental interest in education. The exogenous policy variables in the model are average teacher verbal ability, the proportion of teachers who were education majors in college, the average number of years of teacher experience, the difference in the proportions of black students and black teachers, average class size, the quality of classroom instructional equipment, the extent to which students had been read to before kindergarten and the extent to which teachers are free to adopt classroom procedures adapted to the needs of the students. The remaining exogenous variables in the model are student background variables, a proxy for the prior verbal ability of sixth grade and ninth grade students, and the extent of racial harmony in the school.

The Data

The data set upon which the statistical analyses are based is a statistical random sample of 369 grammar schools and 95 high schools taken from the cross-sectional data collected in 1965 for use in the Coleman Report. The complete Coleman data set was not used because we wished to take account of some of the criticisms of the Coleman Report. The present sample is made smaller than the original sample so that it could be more highly representative of Blacks and cities. In addition, the Coleman figures were subjected to extensive editing to eliminate recording errors, and to ensure internal consistency.

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[1]This variable is omitted from the high school model, where teacher-student relationships are more impersonal.

PART II SUMMARY OF RESULTS, CONCLUSIONS AND RECOMMENDATIONS

RESULTS

The policy variables which the econometric estimates obtained in the model indicate to be important are listed below, in order of descending influence upon student achievement. The effect upon student achievement of the variables listed under headings (1) and (2) is substantial; the influence of variables listed in (3) and (4) is moderate; and the impact of the variables detailed under headings (5) and (6) is quite small, while still being statistically significant. For an understanding of the meaning of the results the reader should consult Chapters I-III of the Analysis and Findings section of the present report.

We find that:

1. Teacher verbal ability is by far the most important determinant of student achievement in both high school and grammar school.

2. Pre-school enrichment programs are the second most important determinants of student achievement with effects lasting at least to the sixth grade.

3. Racial matching of teachers and students is likely to increase student motivation, particularly for black students.

4. Grammar school teachers who were education majors tend to be more effective than other teachers of similar verbal ability in producing student achievement.

The following variables are substantially less important determinants of student achievement than those described above:

5. Class size and the quality of instructional equipment affect the number of positively reinforcing student contacts that can be made by teachers. They are therefore indirect determinants of student motivation.

6. Teacher experience and the relative freedom of teachers to fit instructional techniques to the needs of students influence the extent of classroom disciplinary problems. Because disciplinary problems reduce the time available for positively reinforcing contacts, experience and freedom are indirect determinants of student motivation.

A proxy for the social benefits to be gained from a particular educational policy is the future income stream which would be generated by increasing the level of the pertinent policy variable. In what follows we will refer to the increment in the present value of total future student earnings attributable to a unit increment in a policy variable as the return to that policy variable. This return is calculated assuming that: the rate of discount of future earnings is 5%[2], the increment in lifetime earnings is in the form of a constant yearly sum over a working lifetime of 40 years, the average number of years of education completed by a typical student is 12.8 years and there are 30 students per class. Then: the return to teacher verbal ability[3] on the high school level is $7503 per teacher year, and the return to teacher verbal ability on the grammar school level is $7066 per teacher year. The return to preschool reading programs is $1710. The return to the teacher-student racial difference is $180 for the high school and $283 for the grammar school. The return to teacher education majors is $386. The returns to class size in high schools and grammar schools are $360 and $270 respectively. The return to instructional equipment is $39 and the return to teacher experience is $64. Finally, the return to teacher freedom is $13.

CONCLUSIONS AND RECOMMENDATIONS

The summary of recommendations which follows is stated without the usual caveats concerning the quality of the data, the comprehensiveness of the model, etc. These are to be understood to apply to this as to any other analysis. For a better perspective on the applicability and limitations of the recommendations the reader is urged to examine Chapters I-III of Part III (Findings and Analysis). It should, however, be mentioned even here that the recommendations presented below are the result of a study based on data applying to a variety of schools at a single point of time rather than on panel (time series) data relating to a set of schools over time. Our recommendations therefore do not apply to the transition problems which might be encountered in their implimentation; their validity is for a post-transition period, and describes the average results which might be expected from the changes recommended over a decade or so.

On the basis of our findings we recommend that, if the objective of public education is to enhance the student's academic achievement, their lifetime earning capacity, or their motivation then:

(1) School authorities should alter their salary structures and hiring practices so as to promote the hiring of teachers with greater verbal ability.

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[2]For discount rates of 6%, 7%, and 8% the returns given in the next paragraph should be multiplied by .88, .77, and .69 respectively.

[3]The definitions of the units in which the variables are measured are given in the discussion of Table II of Chapter III of Part III of this report.

(2) On the other hand, emphasis on teacher experience and on semesters of education of teachers appears unwarranted in terms of student educational achievement. Educational results do not justify the current emphasis being placed on these factors in salary structures and hiring practices.

(3) The persistent affect on student achievement of the extent to which students were read to before kindergarten even when one controls for socio-economic background and other home environment and school characteristics suggests that pre-school enrichment programs can be expected to yield large returns, particularly for students whose parents do not provide a verbally active environment.

(4) In assigning teachers to schools, a policy of minimizing teacher-student racial differences should be followed. The importance of the extent of teacher-student racial differences in termining student motivation suggests that because of their roles as meaningful educational models, Black teachers should be preferred to white teachers of <u>equal verbal ability</u> for black students.

(5) For grammar schools, teachers who were education majors in college should be preferred, since they tend to produce students of higher verbal ability than do teachers who have other college majors.

(6) Expenditures for reductions in class size, and improvements in the quality of instructional equipment generate rather small returns in terms of student performance; even though they do affect student motivation.

(7) Reduced class sizes and teacher experience are more important in grammar schools than in high schools.

(8) Teacher freedom to fit classroom procedures to the needs of the students leads to slight improvements in student motivation and achievement. Since an increase in relative teacher freedom is however virtually costless, school authorities might profitably experiment with curriculum designs that allow the teacher more latitude.

PART III FINDINGS AND ANALYSIS

THIS SECTION OF THE REPORT WHICH IS DIVIDED INTO THREE CHAPTERS GIVES THE ANALYTIC AND EMPIRICAL BACKGROUND FOR THE CONCLUSIONS AND RECOMMENDATIONS OF THE STUDY.

Chapter I

THE DETERMINANTS OF STUDENT ACHIEVEMENT IN GRAMMAR SCHOOLS: A SIMULTANEOUS EQUATION APPROACH

I. INTRODUCTION

The purpose of this study is to identify the policy variables that are effective in increasing student verbal achievement in urban grammar schools. Previous studies with the same general goal have had some difficulty in distinguishing effects which are specific to particular policy variables because of the large amount of intercorrelation between variables treated as "independent."

The present paper develops a theoretical model of student motivation that is used as a guide in the specification of an empirical model of student achievement. This empirical model describes the educational process as a chain of causal relationships, thereby greatly reducing the imprecision engendered by multicollinearity. Estimates of the parameters are presented and the policy recommendations resulting from the model are discussed.

The associative theories of learning upon which this paper relies state that student verbal achievement is determined primarily by the number and arrangement of words to which the individual is exposed. In a survey of learning theory, for example, Arthur Jensen says that, "learning verbal labels for objects is greatly facilitated if the labels occur with the objects repeatedly in different verbal contexts."[1] Student classroom exposure to words is, in turn, determined by the ability and training of the teacher and by the willingness of the student to be instructed, student motivation. To understand the learning process, we therefore require a theory of student motivation.

II. THE THEORY OF STUDENT MOTIVATION

The theory of student motivation developed here is essentially an outgrouth of the theory of consumer choice. The latter theory describes how the consumer's dollar budget is allocated among various goods. The present theory describes how the student chooses to allocate his classroom time between academic and non-academic pursuits. In the theory of consumer choice, the consumer is assumed to purchase the satisfaction embodied in the consumption of a bundle of goods and services with the

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[1] "Social Class and Verbal Learning" in Social Class, Race, and Psychological Development; Deutsch, Katz and Jensen editors; New York; Holt, Rinehart and Winston, Inc., 1968, pp. 126-127.

money he spends. Here the student is viewed as purchasing a stream of present and future rewards with the time at his disposal.[2]

It will be assumed that the student divides his classroom time among time spent on academic pursuits and time spent on non-academic pursuits with a view to maximizing the psychological rewards (utilities) resulting from this allocation. The most important rewards associated with time spent on academic pursuits are: parental approval, teacher approval, expected future earnings, and expected future occupational status. The primary rewards associated with time spent on non-academic pursuits are: peer acceptance and leisure (i.e., school time devoted to relief from boring or difficult work). Specific reward items can, of course, appear on both lists. For example, at some schools and in some subgroups of students peer acceptance may depend upon academic status as well. In others, parental approval may be essentially independent of student academic achievement.

In the analysis that follows, the proportion of classroom time spent on academic pursuits M will be viewed as allocated to the purchase of a composite good T, comprising the rewards (teacher acceptance, expected future earnings, etc.) which can be purchased with classroom time devoted to academic work. Similarly, the proportion of time spent on non-academic pursuits ((1) $L = 1 - M$) will be viewed as devoted to the purchase of a composite good P, consisting of the rewards (peer acceptance, leisure) that tend to be acquired by expenditure of classroom time on non-academic endeavors. It will be assumed that the utility function which is maximized by the student is a convex ordinal preference function of T and P, U (T, P). It will also be assumed that the functions describing how academic and non-academic classroom time are transformed into P and T respectively are linear,[3] with (2) $T = a M$ and (3) $P = d L$ and a and d constants.

Given these assumptions it can be shown that the student can transform P into T as described by equation:

$$(4) \qquad T = a - (a/d) P.$$

We can then find the student's desired T and P, and therefore his desired M, by maximizing U (T, P) subject to (4). Using the method of Lagrange we maximize

$$(5) \qquad W = U(T, P) + \lambda [T - a + (a/d) P]$$

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[2]See Gary S. Becker, A Theory of the Allocation of Time, EJ 75: 493-517, Sep. 1965.

[3]It should be stressed that this assumption is made for expositional convenience only and not because it is required in the analysis.

Taking partial derivatives with respect to T and P, setting them equal to zero and eliminating λ we obtain the equilibrium condition

$$(6) \qquad -(a/d) = -(\partial U/\partial P)/(\partial U/\partial T)$$

Equations (4) and (6) are the first order necessary conditions for maximum utility.[4] They may be solved for an equilibrium value of M, M'. M' is the proportion of classroom time the student desires to devote to academic pursuits. It is the theoretical measure of student motivation used in this study.[5]

II. 1. Graphical Analysis

The meanings of these conditions will be explained with the aid of a four-panel diagrammatic analysis based on Figure I. The solid straight lines in panels one through four are representations of equations one through four.

The line a d in the third panel is derived from the other three panels as follows: If the student devotes all his classroom time to academic work he will be at point h in panel 1. This corresponds to a level a of teacher acceptance (see panel 4) and to a zero level of non-academic work. The second panel shows (see point o) that this, in turn, corresponds to a zero level of peer acceptance. Thus point h in panel 1 corresponds to point a in panel 3. Point g in panel 1 corresponds to point o in panel 4, point k in panel 2 and therefore to point d in panel 3. Connecting points a and d by a straight line (representing equation 4) we have the collection of points in (T, P) space that correspond to the line gh in (L, M) space. For example, point n in panel 3 corresponds to point n' in panel 1. Therefore, for the student, choosing a particular mix of teacher and peer acceptance (T', P') is equivalent to choosing the proportion of classroom time M' to be devoted to academic work.

The curved line in panel 3 represents one of the student's indifference curves. We know that the slope of an indifference curve is:

$$dT/dP = -(\partial U/\partial P)/(\partial U/\partial T).$$

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[4]The second order condition for maximum utility is satisfied because of the linearity of equation 4 and the assumed convexity of the student's indifference curves.

[5]This measure is made simple for expositional convenience. A more complete measure would take into account the intensity of work per unit of time that the individual is willing to do.

From equation 4 we know that the slope of the line ad is -(a/d). Therefore, the first order conditions mean that the student should operate at point e in panel 3 where one of his indifference curves is tangent to his transformation curve. This point corresponds to point e' in panel 1. Thus the equilibrium proportion of classroom time devoted to academic work by the student represented in Figure I, would be M'.

III. MOTIVATIONAL CONSEQUENCES OF DIFFERENCES IN UTILITY FUNCTIONS AND TRANSFORMATION FUNCTIONS

We have presented a theoretical model in which the level of student motivation is determined by the student's tastes and opportunities. We will now use the model to investigate the causes of some differences in student motivational characteristics.

III. 1. Motivational Consequences of Differences in Utility Functions

Consider the point r in panel 3. If the student's tangency position were at r rather than at e, he would devote more (M_r rather than M') time to academic work. A tangency position at r indicates that the student finds peer acceptance relatively less important and teacher acceptance relatively more important than a student with a tangency position at e. The nature of family life of low status families (a greater degree of father absence, more mothers who work, and a greater number of siblings with whom to compete for available parental attention) encourages their children to be independent of the family at a relatively early age and to use the peer group as a substitute source of values. These values tend to favor physical prowess and attitudes of independence of authority. By contrast, young middle class children rely primarily on their parents for ego support and development.[6]

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[6]For an excellent summary of social class and racial differences in ego development see, Ausubel and Ausubel, "Ego Development Among Segregated Negro Children," in Education in Depressed Areas; H. Passow, ed.; 1968. In referring to the influence of social class the Ausubels say, "Many of the ecological features of the segregated Negro subculture that impinge on personality development in early childhood are not specific to Negroes as such, but are characteristic of most lower-class populations . . . lower class parents extend less succorant care and relax closely monitored supervision much earlier than their middle-class counterparts. Lower-class children are thus free to roam the neighborhood and join unsupervised play groups at an age when suburban children are still confined to nursery school or to their own backyards. Hence, during the pre-school and early elementary-school years, the lower-class family yields to the peer group much of its role as socializing agent and source of values and derived status. . . This pattern of precocious independence from the family combined with the

(Cont.)

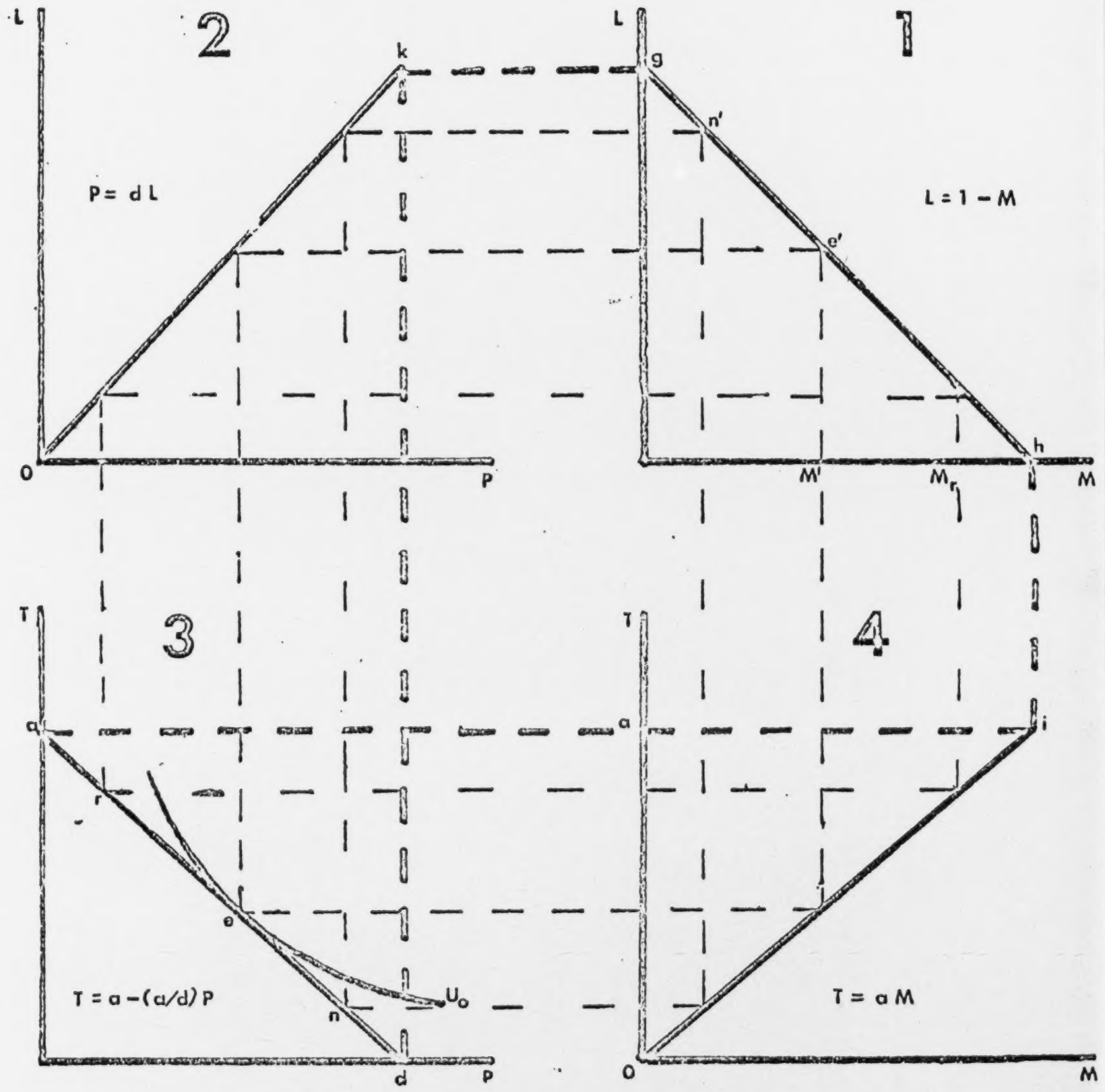

FIGURE I

MOTIVATION EQUILIBRIUM IN GRAMMAR SCHOOL

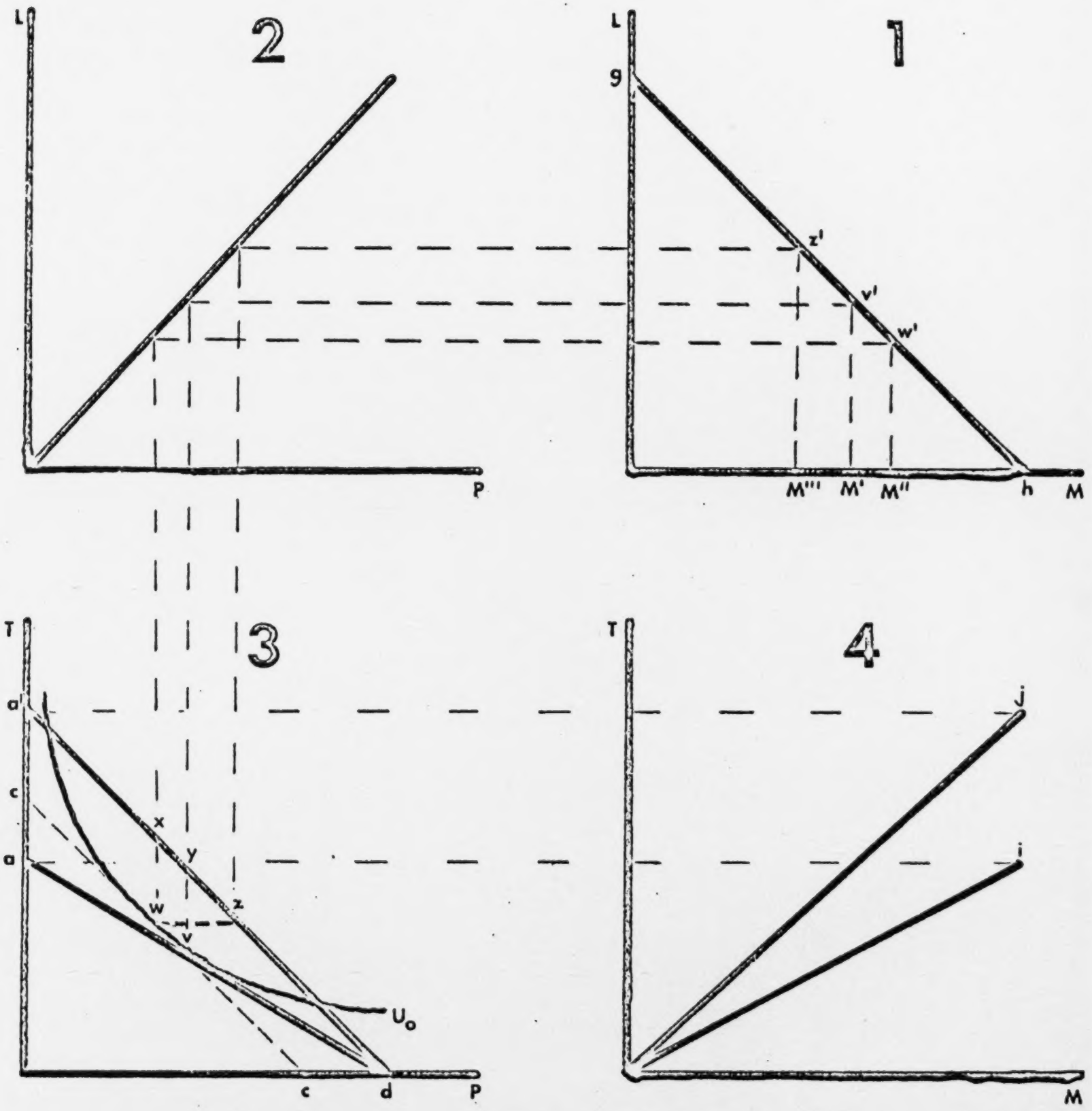

FIGURE II

MOTIVATIONAL CONSEQUENCES OF DIFFERENCES IN THE SECOND EQUATION

Thus we would expect to find relatively many high status students at points like r and relatively many low status students at points like e. The empirically testable version of this proposition for the present study is that, cet. par., low status students are less highly motivated than high status students.

The preceding discussion has dealt with motivational differences that arise from socially induced differences in utility functions. We will now consider motivational differences that arise from differences in opportunity sets. The student's opportunity set is derived from equations one, two, and three. Any differences in opportunity sets must therefore arise from differences in one or more of these equations. Since equation one is the same for all students, whatever opportunity set differences arise stem from equations two and three.

III. 2. Motivational Consequences of Differences in the Second Equation

The second equation indicates the rate at which the student can transform academic work into teacher acceptance. Teacher acceptance for the purposes of this exposition should be thought of as the number of positively reinforcing contacts made by the teacher during the time period under discussion.[7] The rate at which these contacts are made depends upon the teacher's willingness and opportunity.

The teacher's opportunity for contact with the individual student depends in turn upon the amount of attention required of her by other students. This required attention may be expected to rise 1) if the teacher does not have good instructional equipment to constructively occupy her other students; 2) if the teacher must cope with a relatively large number of discipline problems, or 3) if there are a large number of students in the class. Any of these considerations can be expected to reduce the slope of the line representing the rate at which academic time M can be transformed into teacher acceptance T (constant a in equation 2).

The fourth panel in Figure II shows a change in the slope of the second equation. According to our discussion above, a decrease in class size or disciplinary problems or an increase in the quality of instructional equipment can be expected to change the diagram of equation

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[6](Cont.) exaggerated socializing influence of the peer group, although characteristic of both white and Negro lower-class children, does not necessarily prevail among all lower-class minority groups in the United States. Both Puerto Rican and Mexican children enjoy a more closely knit family life marked by more intimate contact between parents and children."

[7]This can be thought of as a school year.

2 from oi to oj. If equation 3 does not change, the change in the second equation will lead to a change in the diagram of equation 4 from ad to a'd.

In panel 3 of Figure II, the student was originally in equilibrium at the tangency point v on indifference curve U_o. This corresponds to a level M' of student motivation (see quadrant 1 in Figure II). In order to show the influence of a change in equation 2 upon student motivation, we will separate the total effect in quadrant 3 into what may be termed an income effect and a substitution effect. By an income effect, we mean the change in consumption of P and T that the student will make as a result of a parallel shift in his opportunity set boundary. By a substitution effect, we mean the change in the student's equilibrium quantities of T and P that will result if the slope of his opportunity set boundary is changed and he is forced to remain on the same indifference curve.

In panel 3 the substitution effect is from v to w. Where the indifference curve U_o is tangent to the dotted line cc which is parallel to the student's opportunity set a'd. The direction of the substitution effect is given by the assumptions about the convexity of the utility function. After changes in the diagram of equation 3 from oi to oj, the student finds that the price of teacher acceptance in terms of time has decreased while the price of peer acceptance has remained the same. Teacher acceptance has therefore become relatively cheaper and peer acceptance has become relatively dearer. The substitution effect from v to w thus results in an increase in T and a decrease in P. In terms of student motivation, the substitution effect involves an increase in motivation from M' to M".

The influence upon student motivation of the income effect resulting from a change in the price of teacher acceptance conflicts with the influence of the substitution effect. The movement from point u on cc to a point on a'd constitutes the income effect. In the theory of consumer behavior a normal good is one whose consumption increases as a result of a parallel shift in the boundary of the opportunity set. An assumption of normality would seem reasonable for both T and P. The meaning of this assumption is as follows: the student will try to increase his consumption of both P and T if his opportunity set boundary shifts out in a parallel manner. This means that the students final equilibrium point will be between x and z. (see panel 3 of Figure II).

If the final equilibrium point were between x and y, a policy like class size reduction would lead to an increase in student motivation. On the other hand, if the final equilibrium point were between y and z, a policy like class size reduction would, perversely, lead to a decrease in student motivation.

If the substitution effect dominates, the association between teacher contact and student motivation estimated empirically will be positive.

However, because of the opposite influences of the substitution and income effects upon student motivation, empirical estimates of the effect of increased teacher contact might be expected to be low.

III. 3. Motivational Consequences of Differences in the Third Equation

The third equation indicates the rate at which the student can transform non-academic classroom time into peer acceptance. For the purposes of this exposition, peer acceptance can be thought of as the number of close friends the student has.

The methods employed by students to transform non-academic classroom time into peer acceptance generally produce disciplinary problems. In fact, for many students, the creation of classroom disruptions is a chief means of acquiring peer acceptance.[8] The teacher who is most sensitive to the process by which disruptions are transformed into peer acceptance can be expected to be most effective in providing countermeasures which blunt the efficiency of that process. It is sometimes said that this kind of teacher sensitivity is acquired through experience or through similarity in teacher-student backgrounds, and is most effective when school regulations do not hinder the implementation of appropriate countermeasures. Thus, we would expect such things as teacher experience, teacher-student racial differences, and relative teacher freedom in classroom organization and techniques of control to affect the slope of the third equation.

The rate at which the student can transform classroom time into peer acceptance also depends upon the values of the peer group. If an attitude of independence is prized, the student will find it easier to buy peer acceptance with a given number of classroom disruptions and more difficult to maintain peer acceptance if he automatically complies with the wishes of the teacher. In speaking of the relationship between student and teacher, Ausubel and Ausubel[9] say, "The lower-class child of school age . . . is coerced by the norms of his peer group against accepting her authority, seeking her approval, or entering into a satellizing relationship with her." Thus we would expect social class to affect the slope of the third equation.

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[8]To loosely test this proposition, twenty students from different classes in two grammar schools were asked two questions: 1) Who is the most popular person in your class? 2) Is he (she) the sort of person who fools around a lot or does he (she) study hard? In every case the most popular person fooled around a lot in class, although several also studied hard. The two schools were polar types; i.e., one was in a poor Black neighborhood and the other was in a well-to-do White neighborhood.

[9]Ausubel and Ausubel, Op. Cit., p. 117.

The second panel in Figure III shows a change in the slope of equation 3. According to our discussion above, a decrease in teacher-student racial differences, an increase in teacher experience, or an increase in teacher freedom will lead to a change in the diagram of equation 3 like that shown from oj to ok. Because of differences in peer group values, students from high status families can be expected to face curves like ok while students from low status families can be expected to face curves like oj. A movement from oj to ok will lead to a change in the diagram in quadrant 3 from ad to ad'.

In panel 3 of Figure III, the student was originally in equilibrium at tangency point v. The total effect upon student motivation arising from a change in the third equation will be separated into an income effect and a substitution effect, as before.

In panel 3, the substitution effect is from v to w. In terms of student motivation, the substitution effect involves an increase from M' to M" (see panel 1 of Figure III). As in the previous case, the direction of the income effect is the opposite of the direction of the substitution effect. The movement from point w on cc (quadrant 3) to a point on ad' constitutes the income effect. If normality is again assumed for both P and T, the student's final equilibrium point will be between x and z.

Thus, if the final equilibrium point were between y and z, an increase in a variable like teacher experience would lead to an increase in student motivation from point M' to a point between M' and M". If the final equilibrium point were between y and x, an increase in a variable like teacher experience would lead to a decrease in motivation from M' to a point between M' and M'''. If the substitution effect dominates the income effect, we should notice a positive but small association between variables like teacher experience and student motivation.

IV. THE EMPIRICAL MODEL OF STUDENT ACHIEVEMENT

The structure of the empirical model to be developed in these pages is anchored in the theory of student motivation formulated above and in an associative theory of learning.

Before proceeding with the specification of the empirical model, however, a few words about the data to be used are in order.

IV. 1. The Data

The data set upon which the following statistical analyses is based is a stratified random sample of 369 grammar schools taken from the cross-sectional data collected in 1965 for use in the Coleman Report.[10]

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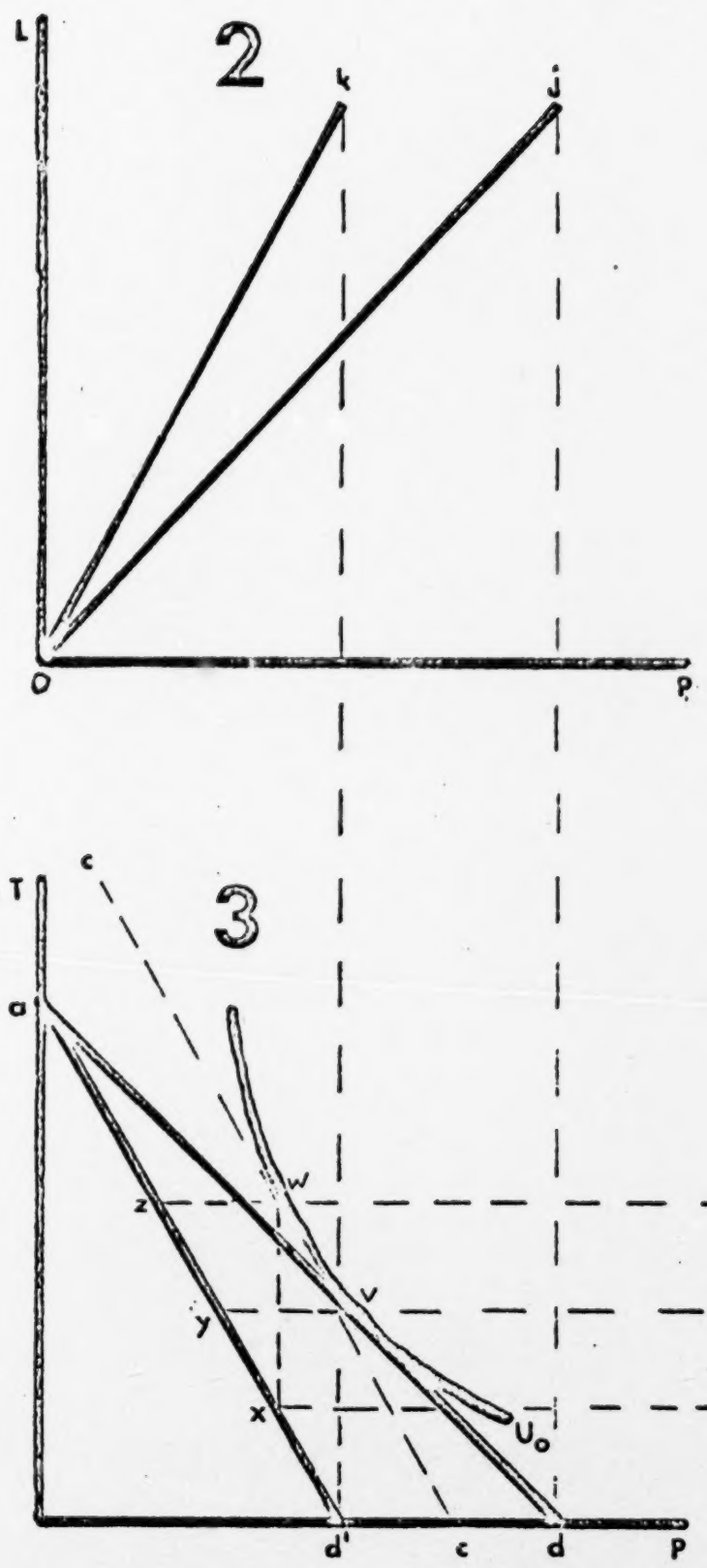
[10] The Report on Equality of Education Opportunity published in 1966 by the Office of Education.



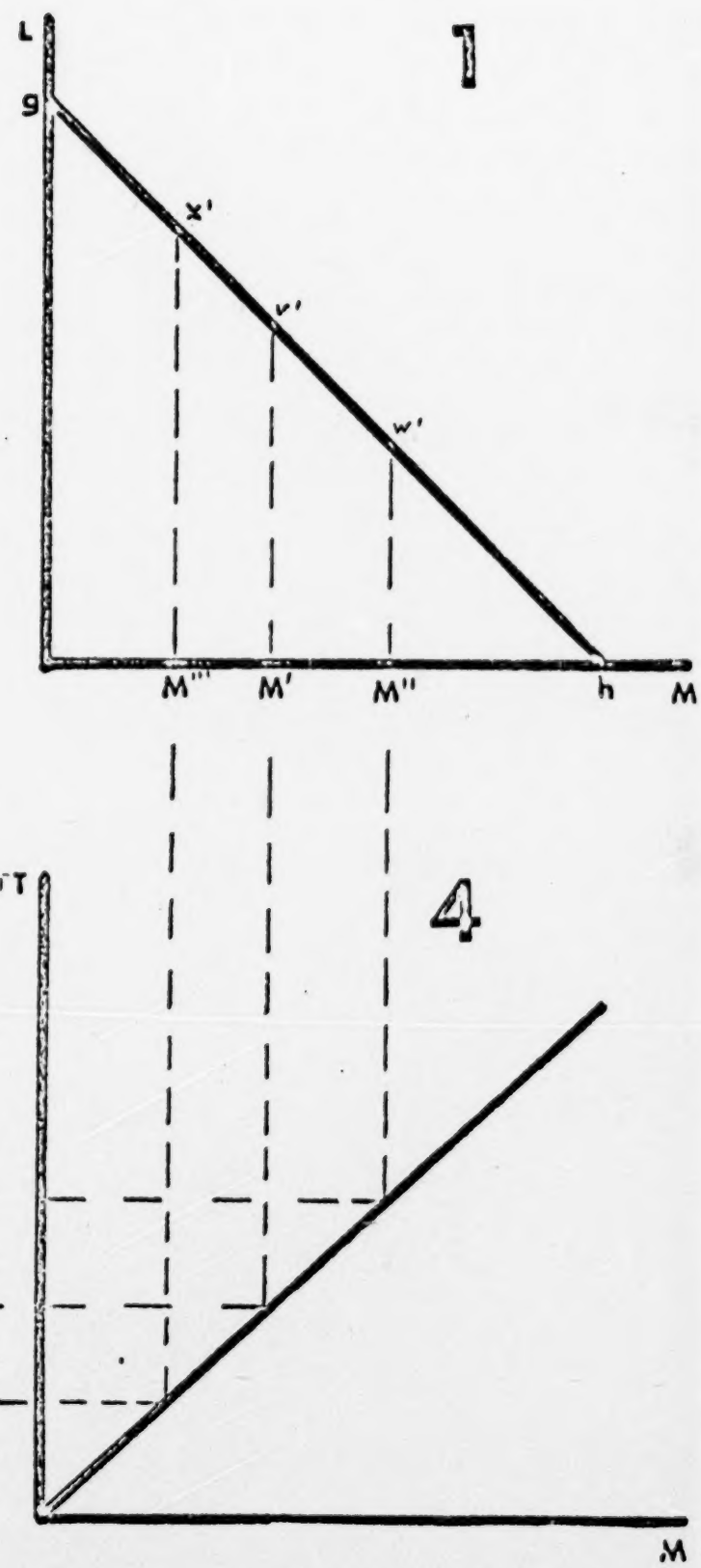
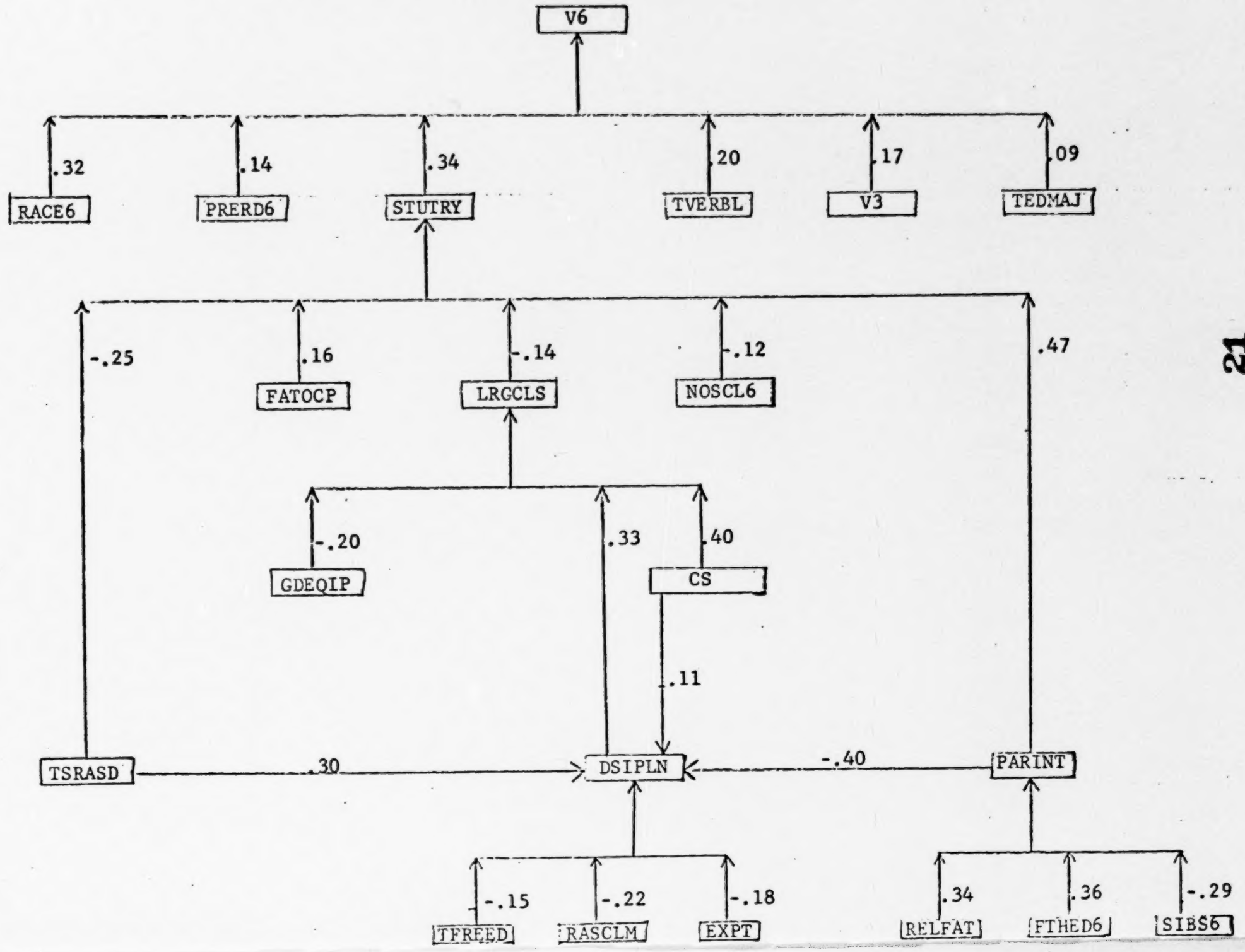


FIGURE III

MOTIVATIONAL CONSEQUENCES OF DIFFERENCES IN THE THIRD EQUATION

FIGURE IV
CAUSATION IN THE GRAMMAR SCHOOL EMPIRICAL MODEL



The complete Coleman data set was not used because we wished to take account of some of the criticisms[11] of the Coleman Report. The present sample is made smaller than the original sample so that it could be more highly representative of Blacks and of cities. To this end, rural schools are eliminated from the sample and the proportion of suburban schools is smaller than in the original sample.

The greatest difference between the original and present samples is that the former consists primarily of individual student, teacher and principal responses to a set of questionnaire items while our data set consists largely of school averages of these responses. These averages were taken principally to facilitate data handling.

IV. 2. Specification and Estimation of the Empirical Model

In the following sections of this paper we will use the theory of student motivation to aid in the specification of a five-equation recursive model of the educational process. Two stage least squares techniques will be used to estimate these equations. Before proceeding with a detailed description of each equation, however, we will first summarize the general flow of causation in the model.

IV. 2.1 Preliminary Summary of the Model

The endogenonous variables in the model are student verbal ability in the sixth grade, student motivation, a proxy for the number of positively reinforcing contacts that can be made by the teacher, student disciplinary problems, and parental interest in education. The exogenous policy variables in the model are average teacher verbal ability, the proportion of teachers who were education majors in college, the average number of years of teacher experience, the difference in the proportions of black students and black teachers, average class size, the quality of classroom instructional equipment, the extent to which students had been read to before kindergarten and the extent to which teachers are free to adopt classroom procedures adapted to the needs of the students. The remaining exogenous variables in the model are student background variables, a proxy for the prior verbal ability of sixth grade students, and the extent of racial harmony in the school.

The general flow of causation in the model can be summarized as follows: In the first equation average student verbal ability in the sixth grade is determined by student motivation and by several other school and background variables. In the second equation student motivation is determined by parental interest, by effective class size (the proportion of teachers who think their classes are too large for effective teaching) and by several other school and background variables.

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[11] See, for example, the excellent analysis by Bowles and Levin in the JHR, III, 1967.

In the third equation, effective class size is determined by the extent of student disciplinary problems, by actual class size, and by the quality of instructional equipment. In the fourth equation, the extent of student disciplinary problems is determined by parental interest in education, by class size, by teacher experience and by several other school variables. In the fifth equation, parental interest in education is determined by a set of socio-economic background variables.

The flow of causation therefore runs from various school and background variables through the extent of disciplinary problems and the opportunity for personal contact between teacher and student, which interact to determine student motivation and therefore student achievement.

IV. 2.2 An Educational Production Function

The educational production function used in this study treats as a dependent variable the average school score (V 6)[12] on a test given to sixth grade students to determine their verbal ability. Without wishing to go into the technicalities of differences between the various learning theories we assume that the school can increase student verbal achievement by manipulating variables affecting the number and arrangement of words to which the individual is exposed so as to achieve a proper exploitation of the network of associations possessed by the student. The kind of mechanism we assume to be at work in this associative theory of learning was nicely summarized by James in 1890 in his, Principles of Psychology. He said of a fact that, "Each of its associates becomes a hook to which it hangs, a means to fish it up by when sunk beneath the surface."

Since the main source of new words in the classroom is the teacher, we would expect on rather straightforward grounds that, cet. par., the average verbal ability of teachers (T V E R B L)[13] would be positively related to V 6. However, the effect of teacher verbal ability upon sixth grade student achievement may be more complicated than this. Highly verbal teachers may be more sensitive to the kind of associations possessed by the students and perhaps better able to alter modes of instruction to fit their needs.

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[12]The numbers and letters in parentheses are the variable names used in the regression equations. V 6 is the average school score on a test given to sixth grade students. Its sample mean and standard deviation are 27.8 and 7.2 respectively.

[13]TVERBL is the average school score on a verbal test given to teachers. Its sample mean and standard deviation are 23.7 and 2.2 respectively.

This latter skill is presumably taught in college courses on elementary education. We would therefore expect V 6 to rise cet. par., as the proportion of teachers who majored in elementary education (TEDMAJ)[14] rose.

As noted in the introductory part of section II, we expect the more highly motivated students (S T U T R Y)[15] to attain higher verbal scores, cet. par., on the grounds that highly motivated students will use their classroom time more effectively and will therefore be exposed to a greater number of association-building learning experiences.

Cultural differences between black and white students might be expected to lead to racial differences in the number and pattern of associations. Therefore, since courses of instruction have primarily been aimed at exploiting the associational patterns of white students, we would expect the average sixth grade verbal score (V 6) to rise as the percentage of white sixth graders (R A C E ^{6})[16] rises. Racial differences in student achievement may also be expected to arise from motivational differences induced by racial differences in socio-economic status.

Like the race variable, the extent to which sixth grade students were read to before school (P R E R D ^{6}) [17] can be expected to alter the number and pattern of associations upon which new knowledge can be based. We would therefore expect the average sixth grade verbal score (V 6) to rise, cet. par., as the value of P R E R D^{6} rises.

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[14]The sample mean and standard deviation of TEDMAJ are .66 and .18 respectively.

[15]STUTRY is a weighted school average of responses given by teachers to the question, "How hard do your students try?" Its values could range from 1 (not very hard) to 4 (very hard) and its sample mean and standard deviation are 2.06 and .63 respectively.

[16]The sample mean and standard deviation of RACE^{6} are 54.61 and 39.26 respectively.

[17]PRERD^{6} is a weighted school average of responses given by sixth grade students to the question, "How often did someone read to you before you started school?" This variable was assigned a value of o if the student answered, "never" and a value 3 if the student answered, "often." The sample mean and standard deviation of PRERD^{6} are 1.65 and .23 respectively.

Finally, we include the third grade verbal test score (V 3)[18] in an attempt to overcome the value added problem. In our study, V 3 is intended to represent the prior verbal ability of the sixth grade students even though V 3 and V 6 are not the test scores of the identical students. This use of V 3 is to some extent justified by the data which indicate that the educational experience and the racial and socio-economic characteristics of third and sixth grade students within a school tend to be very similar.[19]

The equation that follows was estimated by two stage least squares techniques. The numbers preceding the variable names are beta coefficients (the regression coefficients of the standardized variables) and the numbers in parentheses are F statistics (the squares of the t ratios).

$$R^2 = .83 \qquad F = 299.27$$

$$\begin{aligned} V6 = {} & .35\ STUTRY + .32\ RACE^6 + .20\ TVERBL + .17V3 \\ & \quad (93.3) \qquad (76.4 \qquad (54.2) \qquad (32.7) \\ & + .14\ PRERD^6 + .08\ TEDMAJ \\ & \quad (33.3) \qquad (15.6) \end{aligned}$$

The endogenous variables in this equation are V 6 and STUTRY. The exogenous variables are R A C E^6, T V E R B L, V 3, P R E R D^6 and T E D M A J.

IV. 2.3 The Second Equation: Student Motivation

The theory of student motivation suggests that if the goods we have called P and T are normal and the substitution effect tends to dominate the income effect, we should detect a significant influence on motivation from variables affecting the rates at which the student thinks he can transform classroom time into P and T. The theory also suggests that we should expect to find differences in motivation arising from socio-economic differences in utility functions.

It should be recalled that while P and T have for simplicity been called peer and teacher acceptance, they are intended to be indices of goals or goods that can be bought with nonacademic and academic time

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[18] V 3 is the average school score on a verbal test given to third grade students. Its sample mean and standard deviation are 15.76 and 1.85 respectively.

[19] The simple correlations between the background characteristics of third and sixth grade students are on the order of .9.

respectively. Thus, the rate at which the student thinks he can transform classroom time into T depends upon his notions of the returns to academic work in terms of expected future earnings and status as well as teacher and parental approval.

Parental interest in the student's education (P A R I N T)[20] can be expected to influence motivation in two ways. First, the returns to academic work in terms of parental approval are greater if PARINT is greater. Second, PARINT is to some extent an indicator of the nature of the student's family life. Following the argument in section III. 3, we can expect low values of PARINT to be associated with students who tend to value parental acceptance relatively less, and peer acceptance relatively more. High values of PARINT should tend to be associated with students who value parental acceptance relatively more and peer acceptance relatively less. The variable PARINT should therefore be positively associated with student motivation (S T U T R Y).

The perceived returns in future earnings and occupational status will depend in great measure upon the kind of adult models the child has had. The child's estimate of his chances of converting classroom time to a high status occupation will tend, cet. par., to be relatively high if his father's occupational status (F A T O C P)[21] is high and low if his father's occupational status is low.

Similarly, the teacher may function as an effective model if his students can identify with him in the psychological sense. It is assumed that a similar teacher-student racial background facilitates the process of identification. Teachers of a similar racial background will also be more successful in exploiting the pattern of associations of the student, thus motivating him to learn, and in frustrating his attempts to convert non-academic time into peer acceptance, thereby

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[20] PARINT is the proportion of teachers in the school who thought that the parents tended to take an interest in their childrens' school work. The sample mean and standard deviation of PARINT are .48 and .27 respectively.

[21] FATOCP is a weighted school average of sixth grade student responses to a question about their father's occupation. The occupation was given the value o if it carried low status (laborers, etc.) and the value 2 if it carried high status (doctors, etc.). The sample mean and standard deviation of FATOCP are .79 and .29 respectively.

lowering the rate at which L is transformed into P. These effects lead us to expect student motivation to be negatively related, cet. par., to the absolute value of the difference between the percentage of black teachers and the percentage of black students (T S R A S D)[22].

The rates at which the student thinks he can transform classroom time into teacher acceptance (i.e. M into T) and non-academic time into peer acceptance (i.e. L into P) are both affected by the number of reinforcing personal contacts between teacher and student. The variable we have called effective class size (L R G C L S) [23] is used as a proxy for the number of reinforcing contacts. It is expected that L R G C L S will be negatively associated, cet. par., with student motivation, provided the substitution effects dominate the income effects. We would also expect the relevant coefficient to be small.

Finally, the number of schools attended by the average sixth grade student (N O S C L^6)[24] is included in the present list of determinants of student motivation because of research suggesting that personality changes may occur as a result of frequent changes in residence. For example, Glen H. Elder, Jr.[25] says that "Frequent residential changes that introduce discontinuities in the experience of a child may create feelings of insecurity, social isolation and identity confusion." In terms of the model of student motivation this means that the rates at which the student thinks he can transform classroom time into such things as peer or teacher acceptance decline as the number of attended schools increases. Thus the student who has attended many schools may withdraw during classroom time, expending little effort in both academic and non-academic pursuits. N O S C L^6 is, therefore, expected to be negatively associated, cet. par., with student motivation.

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[22]The sample mean and standard deviation of TSRASD are 19.30 and 24.77 respectively.

[23]LRGCLS is the proportion of teachers in the school who thought that their classes were too large for effective teaching. The sample mean and standard deviation of LRGCLS are .50 and .20 respectively.

[24]The sample mean and standard deviation of $NOSCL^6$ are 2.21 and .48 respectively.

[25]Glen H. Elder, Jr., "Socialization of Adolescents" in Borgatta and Lembert (eds.) Handbook of Personality Theory and Research, Rand McNally and Company, Chicago, 1968, p. 243.

The statistics associated with the student motivation equation are as follows:

$R^2 = .68$ $\quad\quad$ $F = 156.39$

$$\begin{aligned} \text{STUTRY} = {} & .47\ \underset{(131.8)}{\text{PARINT}} - .25\ \underset{(60.5)}{\text{TSRASD}} - .14\ \underset{(15.1)}{\text{LRGCLS}} \\ & + .16\ \underset{(18.3)}{\text{FATOCP}} - .12\ \underset{(12.6)}{\text{NOSCL}^6} \end{aligned}$$

In this equation the endogenous variables are STUTRY, PARINT and LRGCLS. The exogenous variables are TSRASD, FATOCP and NOSCL^6.

IV. 2.4 The Third Equation: Effective Class Size

It will be recalled from the discussion in section IV. 2.3 that effective class size (L R G C L S) is used as a proxy for the number of reinforcing personal teacher-student contacts. The number of teacher-student contacts depends upon actual class size (C S), the extent of classroom disciplinary problems (D S I P L N) and the quality of instructional equipment (G D E Q I P)[26]. If instructional equipment is good, if class sizes are low, and if disciplinary problems are minor, the teacher has more time to motivate individual students. Thus we expect DSIPLN and CS to be negatively associated, cet. par., with LRGCLS.

The statistics associated with the teacher-student contact equation are as follows:

$R^2 = .45$ $\quad\quad$ $F = 99.5$

$$\begin{aligned} \text{LRGCLS} = {} & .33\ \underset{(57.0)}{\text{DSIPLN}} + .40\ \underset{(98.5)}{\text{CS}} - .20\ \underset{(20.8)}{\text{GDEQIP}} \end{aligned}$$

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[26] CS is average class size as reported by teachers. The sample mean and standard deviation are 30.3 and 6.2 respectively. DSIPLN is the proportion of teachers in the school who thought that too much time had to be spent on discipline. The sample mean and standard deviation of DSIPLN are .43 and .22 respectively. GDEQIP is the proportion of teachers who thought that their instructional equipment was at least adequate in quality. The sample mean and standard deviation are .7660 and .2208 respectively.

The endogenous variables in this equation are LRGCLS and DSIPLN. The exogenous variables are CS and GDEQIP.

IV. 2.5 The Fourth Equation: Classroom Discipline

The number of classroom disciplinary problems is related to the general level of student motivation in the classroom. Therefore, some of the variables that influence student motivation can be expected to influence the variable DSIPLN. Specifically, the variables PARINT and TSRASD are included in the present equation for many of the same reasons that they were included in the student motivation equation. It is expected that the signs associated with these variables will be the opposite of those in the student motivation equation.

The variable RASCLM[27] is an indicator of the extent of racial harmony in the school. It is expected that RASCLM will be negatively associated, cet. par. with the variable DSIPLN. Supplementary work on this variable not reported in detail here suggests that racial tension increases as the proportion of white students approaches one-half, and that racial tension decreases if effective leadership is provided by the school principal, parental interest in education is high and if the proportion of black teachers closely matches the proportion of black students.

The average number of years of teacher experience (E X P T)[28] is included in the present equation because of the assumption that certain kinds of knowledge and techniques helpful in controlling classrooms are acquired through experience. It is expected that EXPT will be negatively associated with DSIPLN.

The latitude allowed teachers in fitting classroom procedures to the needs of students (T F R E E D)[29] is assumed to be related to the

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[27]The variable RASCLM is the proportion of teachers in the school who thought that racial tension was not excessive. The sample mean and standard deviation of RASCLM are .92 and .10 respectively.

[28]The sample mean and standard deviation of EXPT are 13.47 and 4.78 respectively.

[29]TFREED is the proportion of teachers who thought that they had a reasonable amount of freedom in such matters as textbook selection, curriculum and discipline. The sample mean and standard deviation of TFREED are .76 and .18 respectively.

extent of classroom disciplinary problems. It is expected that TFREED will be negatively related, cet. par., to DSIPLN.

Finally, it is assumed that teachers can deal with disciplinary problems more easily in a small class than in large classes. Therefore, actual class size (CS) is expected to be positively related, cet. par., to DSIPLN.

$R^2 = .56$ $\quad F = 236.60$

$$\underset{}{\text{DSIPLN}} = \underset{(119.5)}{-.39\ \text{PARINT}} + \underset{(66.8)}{.30\ \text{TSRASD}} - \underset{(37.7)}{.22\ \text{RASCLM}}$$

$$- \underset{(25.7)}{.17\ \text{EXPT}} - \underset{(17.2)}{.15\ \text{TFREED}} + \underset{(10.2)}{.11\ \text{CS}}$$

The endogenous variables in this equation are DSIPLN and PARINT. The exogenous variables are TSRASD, RASCLM, EXPT, TFREED and CS.

IV. 2.6 The Fifth Equation: Parental Interest in Education

Parental interest in education (P A R I N T) is assumed to be a function of socio-economic status. Studies such as the one cited in section III. 1 have given results consistent with this assumption.

The variables viewed as primary determinants of parental interest in education are: 1) the proportion of sixth grade students whose real father lives at home (R E L F A T); 2) the average number of years of education attained by the fathers of sixth grade students (F T H E D6); and 3) the average number of children living in the homes of sixth grade students (S I B S6)[30] Relatively high levels of parental interest in education are to be expected in schools where most children live with their real fathers, fathers tend to have relatively many years of education, and where students have few brothers and sisters.

These expectations are consistent with the statistics obtained for the present equation.

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[30]The sample mean and standard deviation of RELFAT are .73 and .16 respectively. The sample mean and standard deviation of FTHED6 are 11.45 and 1.52 respectively. The sample mean and standard deviation of SIBS6 are 3.79 and .73 respectively.

$R^2 = .67$ $F = 243.4$

PARINT = .34 RELFAT (70.7) + .36 FTHED6 (99.4) - .29 SIBS6 (50.5)

The endogenous variable in this equation is PARINT. RELFAT, FTHED6 and SIBS6 are exogenous.

V. SUMMARY AND CONCLUSIONS

The preceding sections of this paper have used the theoretical bases provided by the theory of student motivation and the associative theory of learning to specify a five equation model of the educational process in urban grammar schools. In this final section of the paper we will present a detailed diagram that summarizes the flow of causation in the empirical model and a set of multipliers, derived from the empirical model, showing the relative importance of the educational policy variables in terms of their effects upon student verbal achievement.

In the following diagram the variable names are connected by arrows showing the hypothesized directions of causation in the model. It should be noted that the endogenous variables are those that have arrows pointing to them; all other variables are exogenous. Those exogenous variables that can be manipulated by school authorities are cross hatched.

The numbers at the sides of the arrows are the beta coefficients from the equations in the model. Following Goldberger,[31] "we may think of using the effect on y (the regressand) of a typical or 'equally likely' change in each variable as a measure of importance . . . Now variation in the sample does provide an objective measure of typical changes in the form of the sample standard deviation. This is used in the so-called 'beta coefficients'." These coefficients show the number of standard deviations a regressand will change as a result of a one standard deviation change in a regressor.

The exogenous policy variables in the model may be divided into two groups: 1) those that influence V 6 by their effect on the number and arrangement of words presented to the student and 2) those that influence V 6 indirectly through their effect upon student motivation. Variables from the first group in order of relative importance are TVERBL (beta = .20), PRERD6 (beta = .14) and TEDMAJ (beta = .09). These results suggest that student verbal ability in the sixth grade can be increased by a greater emphasis on pre-school enrichment programs and

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[31] A. S. Goldberger, Econometric Theory, New York, John Wiley and Sons, 1964, p. 197.

by giving preference in hiring to teachers with high verbal ability and to teachers who have taken relatively many college courses in elementary education.

The remaining policy variables influence student verbal ability indirectly through student motivation. We will use the chain rule to compute a set of multipliers showing the effect that each policy variable has on student achievement. Decreasing teacher-student racial differences, TSRASD, can be expected to increase student motivation directly and indirectly through its effect upon classroom disciplinary problems, DSIPLN. The multiplier for the variable TSRASD is therefore computed as follows:

$$m\ (TSRASD) = (-.25)\ (.34) + (.30)\ (.33)\ (-.14)\ (.34) = -.09$$

Decreasing actual class size, CS, increases the teacher's opportunity for personal interaction directly and also by facilitating control of classroom disciplinary problems, DSIPLN. The multiplier for the variable CS is:

$$m\ (CS) = (.40)\ (-.14)\ (.34) + (.11)\ (.33)\ (-.14)\ (.34) = -.02$$

Increasing the quality of instructional equipment, GDEQIP, can increase student motivation indirectly by decreasing effective class size, LRGCLS, and giving the teacher a greater opportunity for personal interaction with her students. The multiplier for the variable GDEQIP is:

$$m\ (GDEQIP) = (-.20)\ (-.14)\ (.34) = .01$$

Increasing teacher freedom in matters of course selection and disciplinary procedures, TFREED, and giving preference in hiring to relatively experienced teachers, EXPT, can be expected to increase student motivation indirectly by decreasing the extent of classroom disciplinary problems. The multipliers for these variables are:

$$m\ (EXPT) = (-.18)\ (.33)\ (-.14)\ (.34) = .003$$

and

$$m\ (TFREED) = (-.15)\ (.33)\ (-.14)\ (.34) = .002$$

Finally, where the political climate permits, school authorities wishing to increase the verbal scores of disadvantaged students might consider bussing them to schools in which disciplinary problems are low because of great parental interest in education, PARINT. Alternatively, highly motivated children might be bussed to problem schools in order to decrease the extent of classroom problems, thereby giving the teacher a greater opportunity for personal interaction with her students. It should, perhaps, be stressed that parental interest in education is determined by socio-economic rather than strictly racial factors.

APPENDIX A

This appendix contains a mathematical treatment of the geometric analysis in sections III.2 and III.3. In these sections it was shown that student motivation would change as a result of opportunity set changes. In the following analysis we will deal with the case in which the change in the opportunity set arises from a change in the rate at which the student can transform classroom time into peer acceptance.

The student's opportunity set is determined by the following equations:

1) $L \equiv 1 - M$

2) $T = aM$

3) $P = dL$

where L is leisure, M is student motivation, and T is teacher acceptance, P is peer acceptance and a and d are constants. The opportunity set boundary in P, T space was written as

4) $T = a - (a/d)P$

This can be rewritten

5) $1 = (1/a)T + (1/d)P$

If a and d are increased in the same proportion c the student's opportunity set will shift in a parallel manner. To take account of the possibility of parallel shifts we can rewrite equation 5 as follows:

6) $1 = (1/ac)T + (1/dc)P$

or

7) $c = (1/a)T + (1/d)P$

The connections between the theory of student motivation and the theory of consumer behavior will be clearer if we adopt the convention that (1/a) is the price of teacher acceptance and (1/d) is the price of peer acceptance. Let

6) $r = (1/a)$

and

9) $s = (1/d)$

then we can rewrite equation 7 as follows:

10) $c = rT + sP$

We assume that the student wishes to maximize U(T, P) subject to equation 10. Using the method of Lagrange we form the equation

11) $G = U(T, P) + \lambda(c - rT - sP)$

Taking first partials and setting them equal to zero we have the first order necessary conditions

12) $G_T = U_T - \lambda r = 0$

13) $G_P = U_P - \lambda s = 0$

and

14) $G_\lambda = c - rT - sP = 0$

We can find the effect upon the student's allocation of time resulting from changes in c, r and s by total differentiation of equations 12, 13 and 14. Allowing all variables to vary simultaneously we have

15) $U_{TT}\, dT + U_{TP}\, dP - r d\lambda = \lambda dr$

16) $U_{PT}\, dT + U_{PP}\, dP - s d\lambda = \lambda ds$

17) $-r\, dT - s\, dP = -dc + Tdr + Pds$

If we regard dr, ds and dc as outside of the student's control (i.e. as constants), we can solve equations 15, 16 and 17 for dT, dP and $d\lambda$, those variables determined by the student. To solve for these variables we first form the bordered Hessian determinant A whose elements are their coefficients in equations 15, 16 and 17.

18) $$A = \begin{vmatrix} U_{TT} & U_{TP} & -r \\ U_{PT} & U_{PP} & -s \\ -r & -s & 0 \end{vmatrix}$$

Replacing the first column of A with the vector of constants on the right sides of equations 15, 16 and 17 we have another determinant

$$19) \qquad B = \begin{vmatrix} \lambda dr & U_{TP} & -r \\ \lambda ds & U_{PP} & -s \\ (-dc + Tdr + Pds) & -s & 0 \end{vmatrix}$$

Solving for dT by Cramer's rule we have

$$20) \qquad dT = B/A$$

Let B_{ij} be the cofactor of the element in the i'th row and the j'th column of B. Expanding according to the first columns of B we have

$$21) \qquad dT = \lambda dr(B_{11}/A) + \lambda ds(B_{21}/A) + (-dc + Tdr + Pds)(B_{31}/A)$$

Letting $dr = dc = 0$ and dividing by ds we have the familiar Slutzky equation

$$22) \qquad (\partial T/\partial s) = \lambda(B_{21}/A) + P(B_{31}/A)$$

If the student is forced to stay on the same indifference when s changes (by a compensating change in c) we know that

$$23) \qquad dU = U_T dT + U_p dP = 0$$

The student will operate at a new tangency position after the change in λ so that we still have

$$24) \qquad U_P/U_T - s/r$$

Therefore

$$25) \qquad rdT - sdP = 0$$

From equation 17 it follows that

$$-dc + Tdr + Pds = 0$$

and from equation 21 we have that the substitution effect of a change in s is

$$26) \qquad (\partial T/\partial s)_{U=U_o} = \lambda(B_{21}/A)$$

We know that λ is positive because it is equal to (U_p/s) which is positive. Expanding B_{21} we have

$$27) \qquad A_{21} = sr$$

sr is also positive. Expanding A we have

28) $$A = U_{TP}\, sr - U_{TT}\, s^2 - U_{PP}\, r^2$$

which was assumed to be positive.* Therefore the substitution effect of a change in s is positive.

Setting dr and ds equal to zero and dividing by dc we have from equation 21 that

29) $$(\partial T/\partial c) = -(B_{31}/A)$$

The assumption that P and T are normal goods is intended to mean that $\partial T/\partial c$ and $\partial P/\partial c$ are positive. Therefore the income effect

30) $$Y_{TS} = P(B_{31}/A) > 0.$$

Relating changes in T to changes in student motivation, we know that the total effect of a change in s can be broken down into a substitution effect and an income effect. The substitution effect upon student motivation is

31) $$(\partial M/\partial s) = (dM/dT) \cdot (\partial T/\partial s)_{U = U_o}$$

$$= r \cdot \lambda (B_{21}/A) > 0$$

This means that student motivation can be expected to rise as a result of a compensated rise in the price of peer acceptance (or an equivalent fall in the rate d at which the student can transform classroom time into peer acceptance).

The income effect of a change in s upon student motivation is

32) $$Y_{MS} = (dM/dT)\, Y_{TS}$$

$$= r \cdot P \cdot (B_{31}/A) > 0$$

This means that student motivation will tend to fall as a result of a drop in apparent income brought about by a rise in the price of peer acceptance.

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* That is, the requirements of the second order conditions for a maximum were assumed in the text to be satisfied.

Chapter II

THE DETERMINANTS OF STUDENT ACHIEVEMENT IN HIGH SCHOOLS: A SIMULTANEOUS EQUATION APPROACH

I. INTRODUCTION

The purpose of this chapter is to identify the policy variables that are effective in increasing student verbal achievement in urban secondary schools. The analysis of the educational process in secondary schools will follow the same pattern as that for the primary schools. As before an associative theory of learning and a theory of student motivation will be used to specify and estimate statistically an empirical model of student verbal achievement in high schools. The empirical model will then be applied to evaluate the relative effectiveness of the educational policy variables.

II. THE THEORY OF STUDENT MOTIVATION IN SECONDARY SCHOOLS

The theory of student motivation developed here is an extension of the theory developed for grammar school students. As in the grammar school case, we assume that the high school student divides his classroom time between academic and non-academic pursuits with a view towards maximizing the psychological rewards resulting from this allocation. As before, the most important potential rewards associated with academic pursuits are parental approval, teacher approval and an increment in expected future status and wealth, and the primary rewards associated with time spent on non-academic pursuits are peer acceptance and leisure (school time devoted to relief from boring or difficult work). However, since an attitude of independence from authority tends to be more highly valued as adolescence proceeds and since the high school student is closer to the job situation, we would expect the high school student to value parental and teacher acceptance relatively less than the grammar school student and to value future job income and status and peer acceptance relatively more. The high school student's desire for teacher approval might be expected to be further weakened by the fact that in secondary schools a given student usually has many teachers and is therefore unlikely to develop as personal a student-teacher relationship as the grammar school student. Compared to the grammar school student, we would also expect the average high school student to be relatively more influenced by his past experiences in school. In particular, it appears reasonable to suppose that the high school student estimates his present set of possible rewards largely on the basis of his past successes and failures in transforming his allocation of classroom time between

academic and nonacademic pursuits into the psychological rewards resulting from this allocation. The major structural differences between the models describing the process of education in primary and in secondary schools therefore arise from differences in the determinants of student motivation.

In the following analysis, the student will be viewed as purchasing a set E of educational benefits with M, the proportion of time he spends on academic pursuits. The educational benefits E are in turn viewed by the student as yielding a composite good Y, composed of such things as future income and status and parental approval. The proportion of class time spent on non-academic pursuits ((1) $L = 1 - M$) will be viewed as devoted to the purchase of a composite good P consisting of peer acceptance and leisure.

It will be assumed that the utility function maximized by the student is a convex ordinal preference function of Y and P, $U(Y, P)$. It will also be assumed that the function describing how academic time is transformed first into E and then into Y and how non-academic time is transformed into P are all linear with zero intercepts. Thus we have the equations (2) $E=hM$, (3) $Y=iE$, and (4) $P=dL$, where h, i and d are all positive constants.

Given equations one through four it can be shown that the student can transform P into Y as described by the equation:

(5) $Y = ih - (ih/d) P$

We can find the student's desired Y and P, and therefore his desired M by maximizing $U(Y, P)$ subject to (5). Using the method of Lagrange we form the function

(6) $W = U(Y, P) + \lambda[Y-ih + (ih/d)P]$

Maximizing with respect to Y and P and eliminating λ we obtain the equilibrium condition

(7) $= (ih/d) = -(\partial U/\partial P/\partial U/\partial Y)$

Equations (5) and (7) are the first order necessary conditions for maximum utility. They may be solved for the equilibrium value of P, P'. With the aid of equation (4) P' implies an equilibrium value of L, L'. As in the grammar school case, L' is to be interpreted as the proportion of classroom time the student wishes to devote to non-academic pursuits.

II. 1. Graphical Analysis

The meanings of the first order conditions will be explained with the aid of the six panel diagram in figure V. The solid straight lines

in panel 1, 2, 3, 5 and 6 are representations of equations 1, 2, 3, 4 and 5 respectively. The 45° line in panel 4 serves the purpose of relating quantities measured on the horizontal axis of panel 3 to quantities measured on the vertical axis of panel 6.

The line td is derived from the other five panels as follows: If the student devote all of his classroom time to academic pursuits he would be at point b in panel 1. This corresponds to a level h of expected achievement (see panel 2) and a zero level of peer acceptance. A level h of expected achievement corresponds to a level ih of incremental expected future income (see point 9 in panel 3, and point r in panel 4). Thus point b in panel 1 corresponds to point t whose coordinates are (0, ih) in panel 6. Point g in panel 1 corresponds to a zero level of expected achievement, a zero level of incremental expected future income and a level d of peer acceptance. Thus point g in panel 1 corresponds to point d in panel 6. Connecting points t and d in panel 6 we have the collection of points in (Y, P) space that correspond to the line gb in (L, M) space. The line td is described by equation 5.

The line U_0 in panel 6 represents one of the student's indifference curves. Its tangency with line td at point e is the point at which the student would achieve maximum utility given his range of opportunities. The corresponding point in panel 1 would be e' with the student willing to devote M' of his classroom time to academic work.

III. MOTIVATIONAL CONSEQUENCES OF DIFFERENCES IN TRANSFORMATION FUNCTIONS

The preceding theoretical model describes how the level of motivation is determined by the student's tastes and opportunities. The analysis of how differences in utility functions and how differences in the rate at which the student can transform classroom time into peer acceptance affect high school student motivation follows the same lines as in the grammar school case, and will therefore not be discussed here. In the present section we will deal with the motivational consequences of differences in the functions that transform classroom time into achievement, and achievement into incremental future income.

III. 1. Motivational Consequences of Differences in the Achievement Function

The second equation, $E = hM$, describes the way in which the student's incremental expected achievement depends upon his academic work. We would expect the student's estimate of his present abilities and potential achievements to be based in large measure on his past test scores and course grades. If the student's past achievement test scores are low (high) we would expect him to have a correspondingly low (high) estimate of the rate, h, at which he can transform classroom time into achievement. Such an approach to estimating h would be rational since

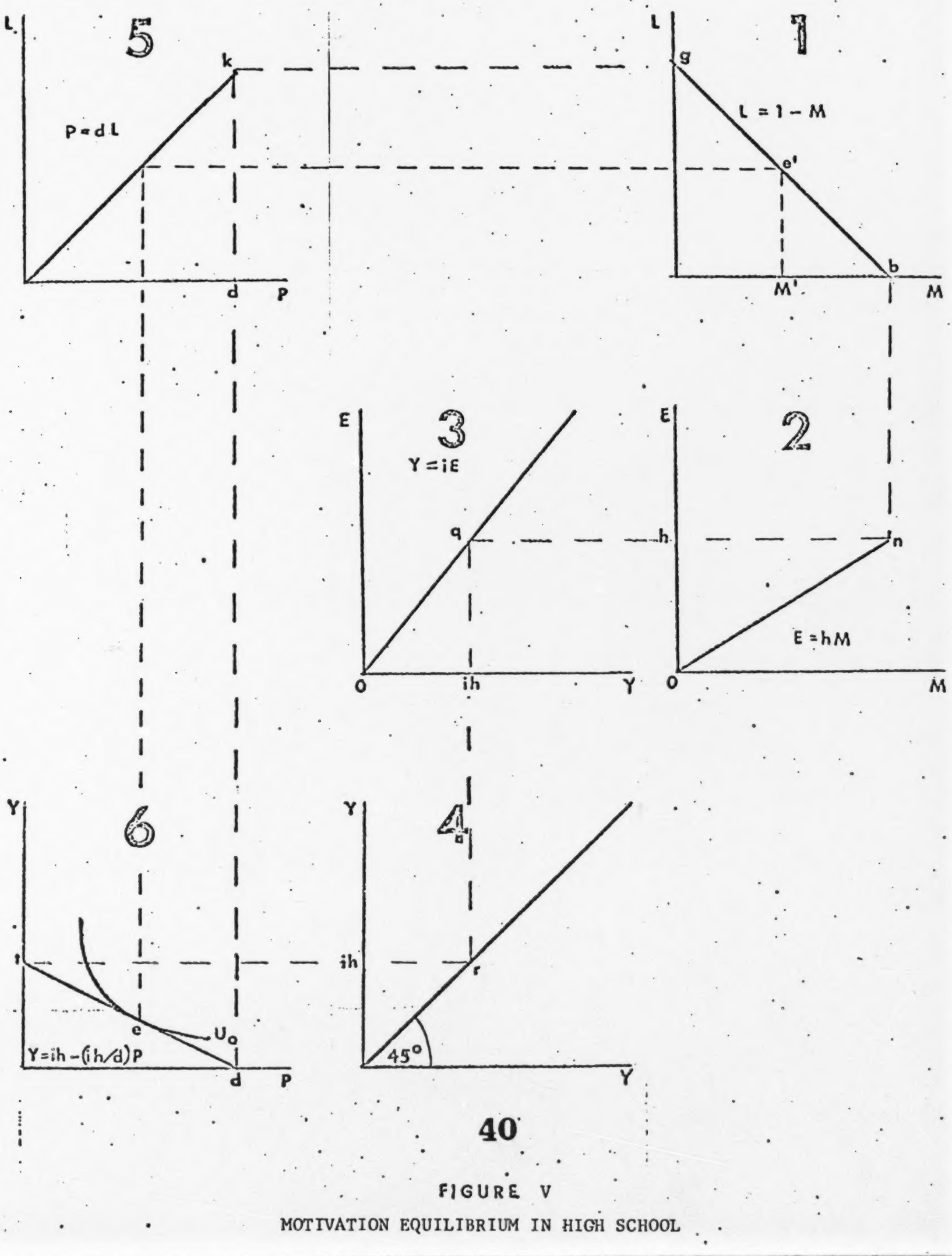

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FIGURE V

MOTIVATION EQUILIBRIUM IN HIGH SCHOOL

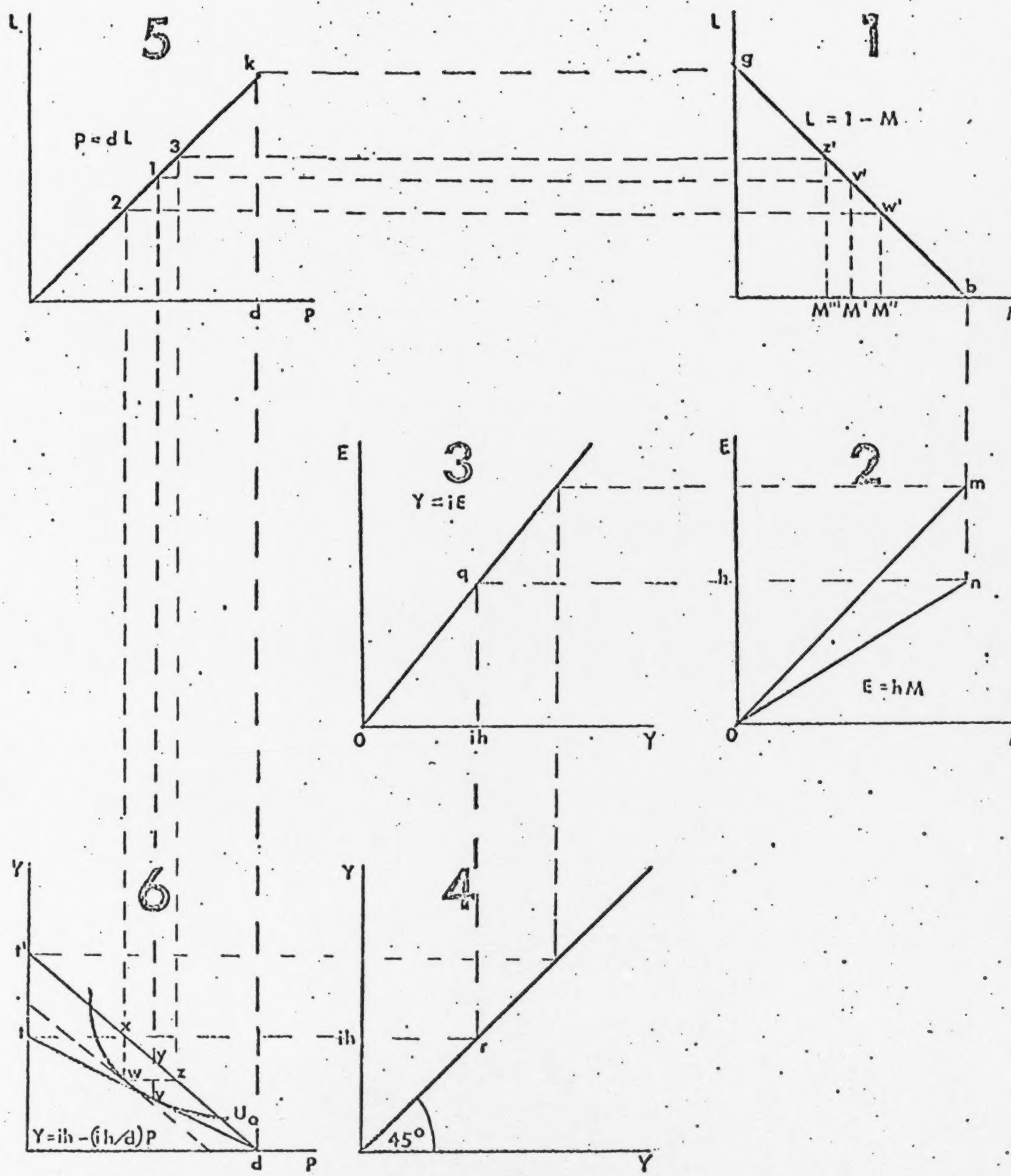

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FIGURE VI

MOTIVATIONAL CONSEQUENCES OF DIFFERENCES IN THE ACHIEVEMENT FUNCTION

the rate at which a student can learn new concepts must depend upon the pattern of associations and level of knowledge he has built up in the past. Thus by manipulating the achievement level of an entering high school freshmen we would expect to affect his estimation of his abilities and consequently, as will be shown, his motivation level while he is in high school.

The lines $\overline{on}$ and $\overline{om}$ in panel 2 of figure VI represent two hypothetical achievement transformation functions for a student. The line on is indicative of a relatively low self-assessment of ability as compared to line om. The opportunity set boundaries corresponding to on and om (assuming all other functions are fixed) are dt and dt' respectively (s-e panel 6). The substitution effect of a change in the achievement function is from v to w in panel 6. Following the dotted lines from points v and w in panel 6 to points 1 and 2 in panel 5 and from there to points v' and w' in panel 2 we see that the substitution effect of the change in the achievement function is from M' to M''. That is the substitution effect of a change in the rate, h, at which the student is able to transform classroom time into incremental achievement is positive. If h increases (decreases), M increases (decreases).

If Y and P are normal goods, (in the economic sense) the income effect of a change in h will be from v to a point on the line between x and z (see panel 6). In terms of student motivation the income effect will be from M'' to a level bounded from below by M'''. Assuming that the substitution effect dominates the income effect, the total motivational effect of a change in h is positive. Thus if achievement levels of entering high school students can be increased we expect high school student motivation to increase.

III. 2. Motivational Consequences of Differences in the Income Function

The third equation, $Y = iE$, describes the way in which the student's incremental expected wealth, Y, depends upon the gain in achievement, E. The rate i at which the student can transform E into Y depends upon the social and economic environment of the student. For example, because of racial prejudice, Blacks expect to earn less than Whites of equal training and ability. Similarly because of family wealth differentials, low status students have a smaller chance of attending college and therefore see a more tenuous connection between achievement and future income than do high status students. In addition, even at similar levels of schooling, children of high status parents obtain higher income and status jobs than do children of low status parents. Thus we would expect black and low status students to have relatively low values of i and white and high status students to have relatively high values of i.

The lines $\overline{oq}$ and $\overline{or}$ in quadrant 3 of figure VII represent two hypothetical income transformation functions for a high school student. If the student faces or he sees a stronger effect of E on Y than he would if he faced oq. The opportunity set boundaries corresponding to oq and or are dt and dt' respectively in panel 6.

Following the analysis in section III. 1, we have that the substitution effect (M" - M' in panel 1) of a change in i is positive; i.e., if i increases (decreases), M will increase (decrease). The income effect will be from M" to a level bounded from below by M"'. Assuming that the substitution effect dominates, the motivational effect of a change in i is positive. This means that if the student is convinced that there is a stronger (weaker) effect of achievement on future income his motivation level will increase (decrease).

IV. THE EMPIRICAL MODEL OF STUDENT ACHIEVEMENT IN HIGH SCHOOLS

In this section we will use the theory of student motivation developed above and an associative theory of learning to aid in the specification of an empirical model of high school student achievement. The data used in the empirical model are discussed and a preliminary summary of causation in the model is presented before the detailed exposition of the model.

IV. 1. The Data

The data set used in the following statistical analysis is a stratified random sample of 95 urban high schools taken from the cross-sectional data collected in 1965 for use in the Coleman report. The complete set of high schools in the Coleman data was not used because 1) we wished to consider only those high schools for which we had information on both the ninth and twelfth grade students, and 2) because we wished to make our sample more representative of black schools and of urban schools than the original sample. The latter was done to overcome some of the criticism made of the Coleman Report. The former was done because we wished to control for the ninth grade verbal test scores in order to overcome the "value added problem" in estimating a twelfth grade educational production function.

As in the grammar school sample, school averages of responses to questionnaire items are used instead of individual student or teacher responses. Again, these averages were taken principally to facilitate data handling and to allow for the possibility of using V9 (the verbal achievement test score of ninth grade classes) to control for prior achievement.

IV. 2. Preliminary Summary of Causation in the Empirical Model

The endogenous variables in the model are student verbal ability in the twelfth grade, high school student motivation, the extent of student disciplinary problems, and parental interest in education. The exogenous policy variables are average teacher verbal ability, average class size and the difference in the proportions of Black teachers and Black students. The remaining exogenous variables in the model are student background variables and a proxy for the prior verbal ability of twelfth grade students.

The general flow of causation in the model may be summarized as follows: In the first equation average student verbal ability in the twelfth grade is determined by student motivation and by several other school and background variables. In the second equation student motivation is determined by the extent of student disciplinary problems and by prior student achievement levels. In the third equation the extent of student disciplinary problems is determined by parental interest in education, class size and two race variables. In the fourth equation parental interest in education is determined by a set of socio-economic variables.

IV. 3. Estimation of the Empirical Model

The four equations comprising the empirical model will be discussed and ordinary least squares estimates of them will be presented in the following pages.

IV. 3.1 An Educational Production Function for Twelfth Grade Students

The dependent variable in this equation is the average school score, V12,[1] on a verbal test given to high school seniors. The explanatory variables in the equation are assumed to affect V12 by their influence on the number and arrangement of words presented to the students in the sample.

The importance of the teacher as the primary source of new words in the classroom leads us to expect teacher verbal ability, TVERBL,[2] to be positively associated with V12 as it was with V6. The socio-economic status of the student is a determinant of the range of his non-classroom exposure to objects and concepts. Therefore we expect an index of socio-economic status, ASSET[3] to be positively associated with V12.

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[1] The mean and standard deviation of V12 are 56.5 and 10.9 respectively.

[2] TVERBL is the average school score on a verbal test given to teachers. Its sample mean and standard deviations are 23.4 and 2.9 respectively.

[3] ASSET is a weighted average of consumer durables in the homes of twelfth grade students. Its mean and standard deviation are 6.8 and .9 respectively.

We expect the more highly motivated students to score higher on verbal achievement tests since they are exposed to a greater number of association-building learning experiences. The index of student motivation used in the grammar school sample was the variable STUTRY, a weighted school average of teacher responses to a question concerning their student's intensity of work. In the high schools, however, the teacher is a less reliable judge of the proportion of classroom time students spend in academic work because 1) each teacher observes a relatively small proportion of the classroom time spent by her students and 2) an attitude of independence of authority becomes more highly valued as adolescence proceeds and actions that facilitate the student's acquisition of learning may have to be masked so that he does not lose peer respect. We have tried to take account of these difficulties by constructing a two component index number, MOTIV, to act as a proxy for student motivation. The first component of MOTIV is based upon teacher's observations of student motivation, STUTRY, while the second is based upon a theoretically important determinant of student motivation, the student's assessment of the importance of education as a determinant of future income, EDLINC.[4]

Finally, we include the ninth grade verbal test score, V9, in an attempt to overcome the value added problem. In this study V9 is intended to represent the prior verbal ability of twelfth grade students, even though V9 and V12 are not the test scores of the same students. This use of V9 is to some extent justified by the data which indicates that the educational experience and the socio-economic and racial characteristics of ninth and twelfth grade students within a school tend to be very similar.

The equation that follows was estimated by ordinary least squares techniques. The numbers preceding the variable names are beta coefficients (the regression coefficients of the standardized variables) and the numbers in parentheses are F statistics (the squares of the t ratios).

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[4]The proxy MOTIV was computed according to the equation MOTIV = I + J, where I and J are the normalized values of STUTRY and EDLINC. STUTRY, as in the grammar school analysis, is a weighted school average of responses given by teachers to the question, "How hard do you students try?" Its values could range from 1 (not very hard) to 4 (very hard). The variable EDLINC is the proportion of twelfth grade students who disagreed with the statement, "Even with a good education, I'll have a hard time getting the right kind of job."

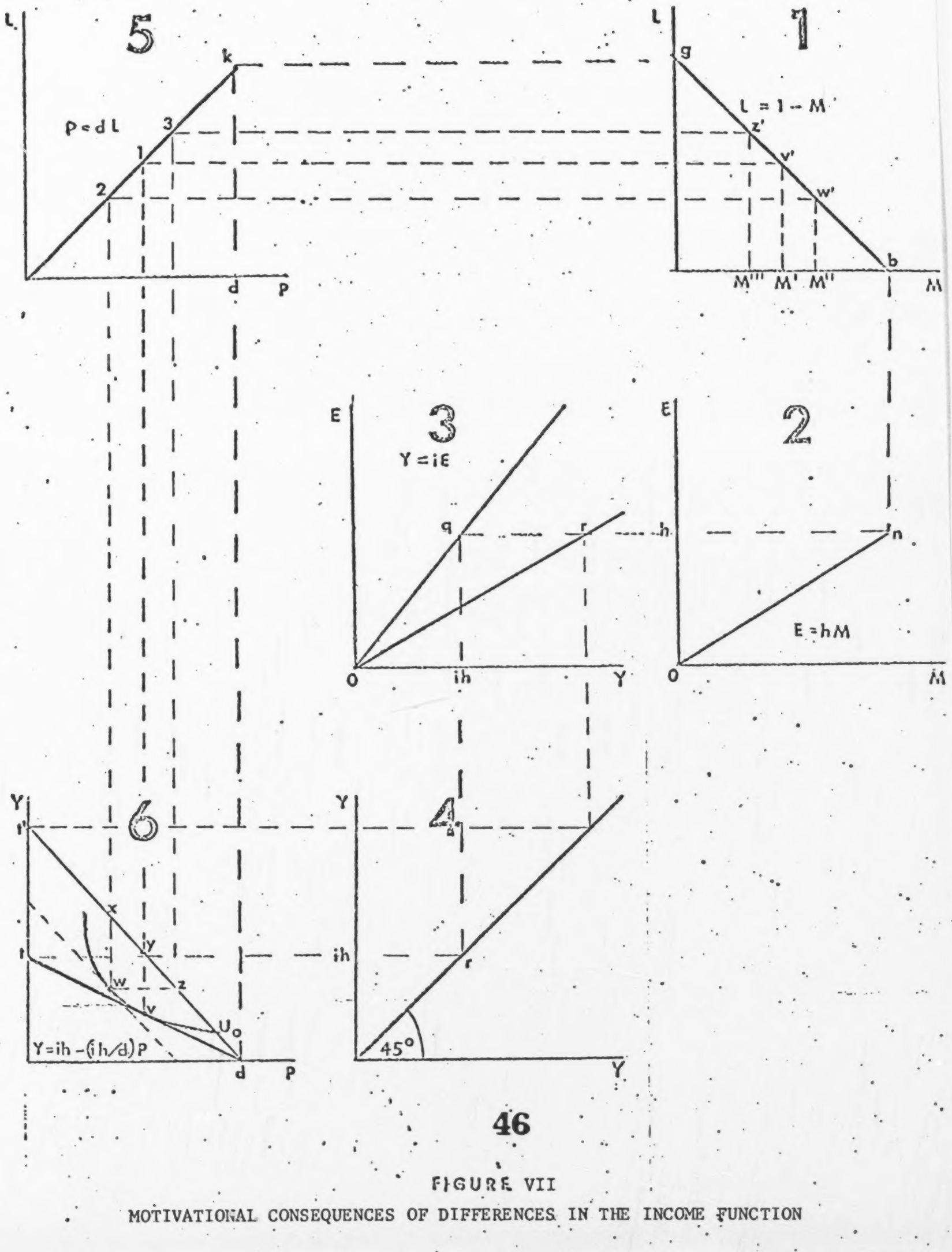

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FIGURE VII

MOTIVATIONAL CONSEQUENCES OF DIFFERENCES IN THE INCOME FUNCTION

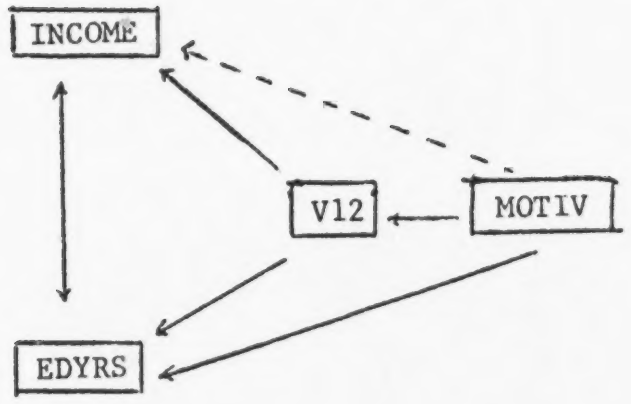

FIGURE VIII

DETERMINANTS OF FUTURE INCOME

$R^2 = .94$

$$V12 = \underset{(96.8)}{.59\ V9} + \underset{(20.6)}{.21\ ASSET} + .16\ TVERBL + \underset{(6.7)}{.10\ MOTIV}$$

IV. 3.2 The Second Equation: Student Motivation

The theory of student motivation suggests that under certain conditions we should detect a significant influence on motivation from variables affecting the rates at which the student thinks he can transform classroom time into P and Y. In section III. 1. of the present chapter we suggested that the rate h at which the student can transform classroom time into incremental achievement depends upon his past level of achievement. This implies that, ceteris pasibus, an increase in past achievement should lead to an increase in the rate at which the student can transform classroom time into incremental income[5] and a consequent (see figure 4) increase in motivation. V9 is therefore expected to be positively associated with the proxy for student motivation, MOTIV.

Classroom disciplinary problems are indicative of peer pressures towards non-academic uses of class time. Disciplinary problems can also reduce the academic classroom time available to students by increasing the proportion of time that the teacher has to devote to disciplinary countermeasures. We would therefore expect the extent of classroom disciplinary problems, DSIPLN,[6] to be negatively related to the level of student motivation.

The statistics associated with the student motivation equation are as follows:

$R^2 = .57$ $\qquad$ $F = 60.4$

$$MOTIV = \underset{(72.0)}{.65\ V9} - \underset{(6.5)}{.20\ DSIPLN}$$

IV. 3.3 The Third Equation: Disciplinary Problems

The theory of student motivation suggests that the rates at which the student can convert classroom time into peer approval and into future income or parental approval simultaneously determine the level of student motivation and the extent of student disciplinary problems. If the

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[5]It also implies that future researchers in this area may expect to have some success with polynomial curve fitting techniques in production function estimation.

[6]DSIPLN is the proportion of teachers who though that too much time had to be spent on discipline. The mean and standard deviation of DSIPLN in the high school sample are .27 and .20 respectively.

student expects a relatively small payoff for achievement in terms of future income he can be expected to devote relatively more time to classroom disruptions with a view toward acquiring peer approval and relatively less time to serious academic work. Thus we expect Black to devote relatively more time than Whites to classroom disruptions because Blacks generally earn less than Whites of equal education and ability because, in view of financial constraints, they are less likely to use academic achievement to gain admission to college, and because for blacks non-academic use of time is the primary means of gaining peer approval. For these reasons we expect the proportion of white students in the twelfth grade, RACE12,[7] to be negatively related to DSIPLN.

Parental interest in education provides parental reinforcement of academic effort. We therefore expect parental interest in education, PARINT,[8] to be negatively related to DSIPLN.

The variables TSRASD,[9] the difference in the proportions of white teachers and students, and CS,[10] class size, are included in this equation because they affect the rate at which the student can expect to transform classroom disruptions into peer approval. TSRASD is intended to indicate teacher-student background similarities and therefore the sensitivity of the teacher to the way in which disruptions are transformed into peer acceptance. It is assumed that this kind of sensitivity is helpful in suggesting appropriate countermeasures. Therefore TSRASD is expected to be negatively related to DSIPLN.

Finally, it is assumed that teachers can deal with disciplinary problems more easily in a small class than in a large class. Therefore CS is expected to be positively related to DSIPLN.

The statistics associated with the third equation are as follows.

$R^2 = .42$ $\qquad\qquad$ $F = 16.2$

$$\underset{\qquad}{\text{DSIPLN}} = \underset{(8.4)}{-.31\ \text{RACE12}} \underset{(7.5)}{- .29\ \text{PARINT}} + \underset{(6.0)}{.21\ \text{TSRASD}} + \underset{(4.7)}{.18\text{CS}}$$

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[7]The mean and standard deviation of RACE12 are .56 and .43 respectively.

[8]The mean and standard deviation of PARINT are .31 and .24 respectively. PARINT is the proportion of teachers in the school who thought that parental interest in education was not lacking.

[9]The mean and standard deviation of TSRASD are .09 and .18 respectively.

[10]The mean and standard deviation of CS are 29.00 and 7.76 respectively.

IV. 3.4 The Fourth Equation: Parental Interest in Education

Parental interest in education is primarily determined by social class. Thus the variables included in this equation are 1) the average number of years of education attained by the fathers of twelfth grade students, FTED12;[11] and 2) the proportion of twelfth grade students whose real father lives at home, RELFAT.[12] It is expected that FTED12 and RELFAT will both be positively associated with PARINT.

The statistics associated with the fourth equation are:

$R^2 = .52$ $\qquad\qquad$ $F = 49.3$

$$\underset{}{\text{PARINT}} = \underset{(50.5)}{.54\ \text{FTED12}} + \underset{(22.2)}{.36\ \text{RELFAT}}$$

V. SUMMARY AND CONCLUSIONS

There are only three school controlled policy variables that influence student verbal ability in high schools, either directly or indirectly through student motivation. Student verbal ability in the twelfth grade V12 can be increased directly, by giving preference in hiring to teachers with higher verbal ability. The multiplier for TVERBL=.16.

The remaining policy variables affect student achievement in the twelfth grade through their impact on student motivation. Decreasing teacher-student racial differences, TSRASD, can be expected to increase student motivation through its effect upon classroom disciplinary problems, DSIPLN. The multiplier for TSRASD is:

$$m\ (\text{TSRASD}) = (-.2)(.10)(.21) = -.004$$

Decreasing actual class size, CS, facilitates control of classroom disciplinary problems, DSIPLN. The multiplier for CS is

$$m\ (\text{CS}) = (-.2)(.10)(.18) = -.0036$$

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[11] The mean and standard deviation of FTED12 are 9.9 and 1.5 respectively.

[12] The mean and standard deviation of RELFAT are .74 and .13 respectively.

Chapter III

EDUCATIONAL INPUTS AND THEIR RELATIONSHIP TO FUTURE STUDENT EARNINGS

I. INTRODUCTION

In the preceding chapters of this paper we have investigated the relationships between a set of educational inputs and outputs. We have shown that student achievement and motivation are determined in a recursive process that starts with the student's home life and continues with the home, peer, and school influences that impinge upon the student as he goes through grammar school and high school. The interest of economists in the process of education stems from studies that show a relationship between completed years of education and earnings.[1] In this chapter we will show how these income-education studies relate to the model presented in the previous chapters. This connection will be established by (1) a discussion of the immediate determinants of the levels of schooling and earnings; and (2) a summary of the grammar school and high school submodels showing their relationship with each other and with the expected levels of schooling and future income.

II. THE DETERMINANTS OF SCHOOLING AND INCOME

In this section we will show that the expected level of schooling depends upon variables that are determined in our high school model. After this, several earnings studies will be examined in the light of our findings in an effort to specify the causal links between the educational process and future earnings.

II. 1. The Direct Determinants of Schooling

The equation presented in this subsection treats as a dependent variable the number of years of education the average student is expected to complete, EDYRS.[2] EDYRS can be expected to increase if there are fewer high school dropouts and/or more students going on to college.

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[1]See, for example, the important paper by Zvi Griliches, "Notes on the Role of Education in Production Functions and Growth Accounting," in Education, Income, and Human Capital, W. Lee Hansen ed., Published by the NBER in 1970.

[2]The sample mean and standard deviation of EDYRS are 12.8 and .82 respectively. EDYRS is actually a proxy for average completed years of education since the Coleman data upon which this study is based do not contain follow-through information on its students. They do, however, contain estimates of

The number of years of education the student is willing to complete depends upon the relative values of the rewards he can expect to reap by staying in school as opposed to leaving. The primary rewards associated with school attendance are increased future earnings and status and present parental approval. The primary rewards associated with leaving school are increased present earnings and freedom from the restriction of movement and submission to authority that are usually required in schools.

These reward sets are similar to those confronting the student when he decides what proportion of his time to devote to academic pursuits. Thus the forces that determine student motivation can be expected to determine EDYRS, We will use our measure of student motivation, MOTIV[3], as a proxy for these forces. We expect MOTIV to be positively related to EDYRS.

The student's decision on whether or not to go to college depends upon the willingness of colleges to accept him as well as his willingness to go. College admission requirements (the use of the SAT and similar tests) indicate that colleges are more willing to accept students who score higher on achievement tests. This means that search costs and fear of rejection will be lower for students of high ability. Thus we expect the average school score on a twelfth grade verbal test, V12[4], to be positively related to EDYRS.

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2 (cont) the dropout rates within high schools and of the proportion of high school graduates who go on to college. The scaling for each of the categories was as follows: 10 years for high school dropouts; 12 years for high school graduates who don't go on to college; and 14 years for those who go to college. Let PD be the proportion of students who drop out of high school, and let PC be the proportion of students who go to college. EDYRS was computed according to the formula: $EDYRS = 10 \cdot PD + (1 - PD)\,[12(1 - PC) + 14 \cdot PC]$.

[3] See the previous chapter for a full discussion of the determinants of MOTIV.

[4] The mean and standard deviation of V12 are 56.3 and 11.0 respectively.

The equation that follows was estimated by ordinary least squares techniques. The numbers preceeding the variable names are beta coefficients and the numbers in parentheses are F statistics (the squares of the t ratios).

$R^2 = .45$ $\quad\quad$ $F = 39.4$

$$\underset{}{\text{EDYRS}} = \underset{(18.1)}{.48\ \text{V12}} + \underset{(4.1)}{.23\ \text{MOTIV}}$$

II. 2. The Determinants of Future Income

The Griliches study cited in section I of this chapter contains the following regression equation:[5]

$R^2 = .836$

$$\log Y = 8.938 + \underset{(.007)}{.051S} + \underset{(.0009)}{.0042A}$$

where Y is income at age 35, S is years of school completed, A is IQ at age 10 and the numbers in parentheses are standard errors. The sample over which the regression is run is taken from a group in Malmo, Sweden.

Both S and A seem to be significant predictors of log Y. In terms of our model, S is similar[6] to the variable EDYRS. The variable A is presumably to be taken as a proxy for the ability of the subjects at ages higher than 10. In terms of our model, however, ability at age 14 (V9, which we suppose is strongly influenced by earlier ability) is an important determinant of motivation (MOTIV) as well as later ability (V12). Thus the influence of A may be due to its role as a proxy for motivation. This view is consistent with the point made by Conlisk[7] that the importance of motivation may have been understressed in studies relating income education. Unfortunately, the available

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[5]See page 98 in Hansen, op. cit.

[6]That is, S is the number of years of education completed by individuals whereas EDYRS is the number of years of education we expect the average student in a high school to complete.

[7]See the interesting comment by John Conlisk in Hansen, op. cit., on pages 122-123.

American data do not permit a good test of the hypothesis that motivation exerts an independent influence on income. Another modification of the relationship between ability and earnings has been suggested by John Hause[8] in a recent issue of the American Economic Review. In that article, Hause argues that an interaction between schooling and ability is important in the determination of earnings; i.e., that the payoff to ability is greater for persons of higher educational attainment.

We cite these studies in an effort to specify the causal links between the educational process and future earnings. The arrows in Figure VIII indicate the hypothesized directions of causation. Thus, V12 and EDYRS are both viewed as determinants of INCOME while the dashed arrow indicates the tentative nature of the hypothesis that motivation directly affects INCOME.

III. SUMMARY OF THE COMPLETE MODEL

Past chapters in this study have shown how achievement and motivation are determined in grammar schools and high schools. The previous section of this chapter showed how achievement and motivation are related to years of education and future earnings. In the present section we will present a diagram that summarizes the flow of causation in the process of education from the grammer school years, through high school and the student's decision about the number of years he will attend school. Using this diagram we will trace the effects of changes in a policy variables through the model.

The diagram labelled Figure IX contains the names of all the variables used in this study. The arrows connecting these variable names indicate hypothesized directions of causation. For the sake of clarity we have bracketed the grammar school and high school submodels, labelling them I and II respectively. In the following pages we will summarize the grammar school and high school models separately and then discuss the relative effectiveness of the policy variables in them.

III. 1. Submodel I: The Grammar School

Submodel I is concerned with the determination of student verbal ability in the sixth grade. In the grammar school, student verbal ability (V6) is determined by student motivation (STUTRY) and several other school and background variables. Student motivation is

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[8] John C. Hause, "Ability and Schooling as Determinants of Lifetime Earnings, or If You're So Smart Why Aren't You Rich?", AER, May 1971, p. 289.

determined by effective class size (LRGCLS) and several other school and background variables. The major components of effective class size are the quality of instructional equipment (GDEQIP), actual class size (CS), and the extent of disciplinary problems (DSIPLN). DSIPLN is, in turn, determined by parental interest in education and several school variables.

Ignoring the possibility of bussing (i.e., using bussing to increase the value of PARINT, thereby decreasing the value of DSIPLN) there are eight policy variables in the grammar school submodel. Those that directly influence V6 are the extent of preschool reading (PRERD[6]), the proportion of teachers who were education majors in college (TEDMAJ) and the average verbal ability of teachers (TVERBL). The teacher-student racial difference (TSRASD) influences V6 through its direct and indirect (through DSIPLN) effects upon student motivation. The quality of instructional equipment (GDEQIP) and actual class size (CS) affect student motivation through effective class size. Class size and teacher freedom (TFREED) and experience (EXPT) affect motivation and V6 indirectly through their influence on DSIPLN.

III. 2. Submodel II: The High School

Submodel II is concerned with the determination of years of education (EDYRS), twelfth grade verbal achievement (V12), and high school student motivation (MOTIV). Years of education is determined by motivation and twelfth grade verbal ability. Twelfth grade verbal ability is determined by high school student motivation, prior verbal ability (V9), and several other variables. High school student motivation is determined by prior verbal ability and a set of school and background variables whose influence is summarized by the extent of disciplinary problems (DSIPLN).

There are three policy variables in the high school submodel: teacher-student racial difference (TSRASD), class size (CS), and teacher verbal ability (TVERBL). The first two variables affect the values of MOTIV through their effect upon the variable DSIPLN. They affect the values of V12 and EDYRS indirectly through their effect upon MOTIV. The variable TVERBL affects V12 directly and EDYRS indirectly.

III. 3. The Policy Variables

The relative effectiveness of a policy variable must be defined in terms of a criterion variable or objective function. We will first use EDYRS as a criterion variable and then discuss the relationship between years of education and student future income.

III. 3. 1 Relationship Between Educational Inputs and Years of Education

Relative effectiveness will be defined as the multiplier that indicates the number of standard deviations expected years of education will change as a result of one standard deviation change in a policy variable. These multipliers can be computed very easily in the high school case because we have equations that describe all the paths between the high school policy variables and EDYRS.

The estimation of the multipliers for the grammar school variables is complicated by the fact that because there were no schools in our sample that had both a sixth grade and a ninth grade we were unable to estimate an equation that describes the influence of V6 upon V9. However, an approximation of this relationship can be used to estimate the relative effectiveness of the grammar school policy variables. Let us suppose that the achievement level at the end of junior high school (V9) is a function of the achievement level at the beginning of junior high school (V6) and the level of motivation in junior high school (MOTIVJ). In addition let us suppose that MOTIVJ depends upon V6. The equations describing these relationships can be written as follows:

$$V9 = a\ V6 + b\ MOTIVJ$$

and

$$MOTIVJ = c\ V6$$

where a, b, and c represent constants.[9] By substitution we obtain:

$$V9 = (a + bc)\ V6$$

The quantity (a + bc) represents the sum of the direct and indirect effects of V6 upon V9. The impact of grammar school variable X upon expected years of education will then be

$$m(EDYRS) = \frac{\partial V6}{\partial X} \cdot (a + bc) \cdot \frac{\partial EDYRS}{\partial V9} .$$

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[9]We assume that the variables have been standardized and that a, b, and c, therefore, have the dimensionality of beta coefficients.

In the following pages we will present a set of tables in which alternative assumptions about the magnitude of (a + bc) will be used to provide estimates of the relative effectiveness of each of the grammar school policy variables. The range of values taken on by (a + bc) was arrived at by examining the influence of prior verbal ability upon subsequent verbal ability in grammar schools and high schools. In the grammar school case, the simple correlation between V3 and V6 is .71 implying an upper limit for (a + bc) of .7 and in the high school case the simple correlation between V9 and V12 is .95. The range considered for (a + bc) therefore is from .4 to .9.

III. 3. 2 Table I: Schooling Multipliers

Table 1 contains a set of multipliers that indicate the number of standard deviations of EDYRS (expected years of education) will change as a result of a one standard deviation change in a policy variable. These multipliers are computed on the basis of the interconnections depicted in Figure IX, taking account of both direct and indirect effects of each of the policy variables. The first row of the table contains the values assumed for (a + bc). The first column contains the variable names, together with a designation indicating whether the respective variables are derived from the high school or grammar school submodels.

The entries in the table indicate the effect, in standard deviation units, of a one standard deviation change in each instrument variable. Both the policy variables and their effectiveness are measured in standard deviation units, in order to facilitate comparison among variables. As indicated earlier, one may think of a one-standard-deviation change in a given variable to be of roughly the same probability as that of another variable. In general, the multipliers are listed in order of size with the larger multipliers coming first. Interestingly enough, the rank order of the multipliers seem to be largely unaffected by the values assumed for (a + bc).

The meaning of the table will be illustrated by a consideration of the multipliers appearing in the second row and corresponding to the variable TVERBL-G (the average verbal ability of grammar school teachers). The multiplier of TVGRBL-G that corresponds to a + bc = .4 is given by the entry .037 that appears in the second row, first column. This number may be interpreted as meaning that if the average verbal ability of teachers in a grammar school is increased by one standard deviation (i.e., by 2.2 points on the verbal test given to teachers), the children in that grammar school can be expected to add (.037) (.8146) = .029 years to their education, where .8146 is the standard deviation of EDYRS. If (a + bc) is taken to be .5 a one standard deviation increase in TVERBL-G will lead to an expected increase of .046 standard deviations of EDYRS, or .046 x .81 = .037

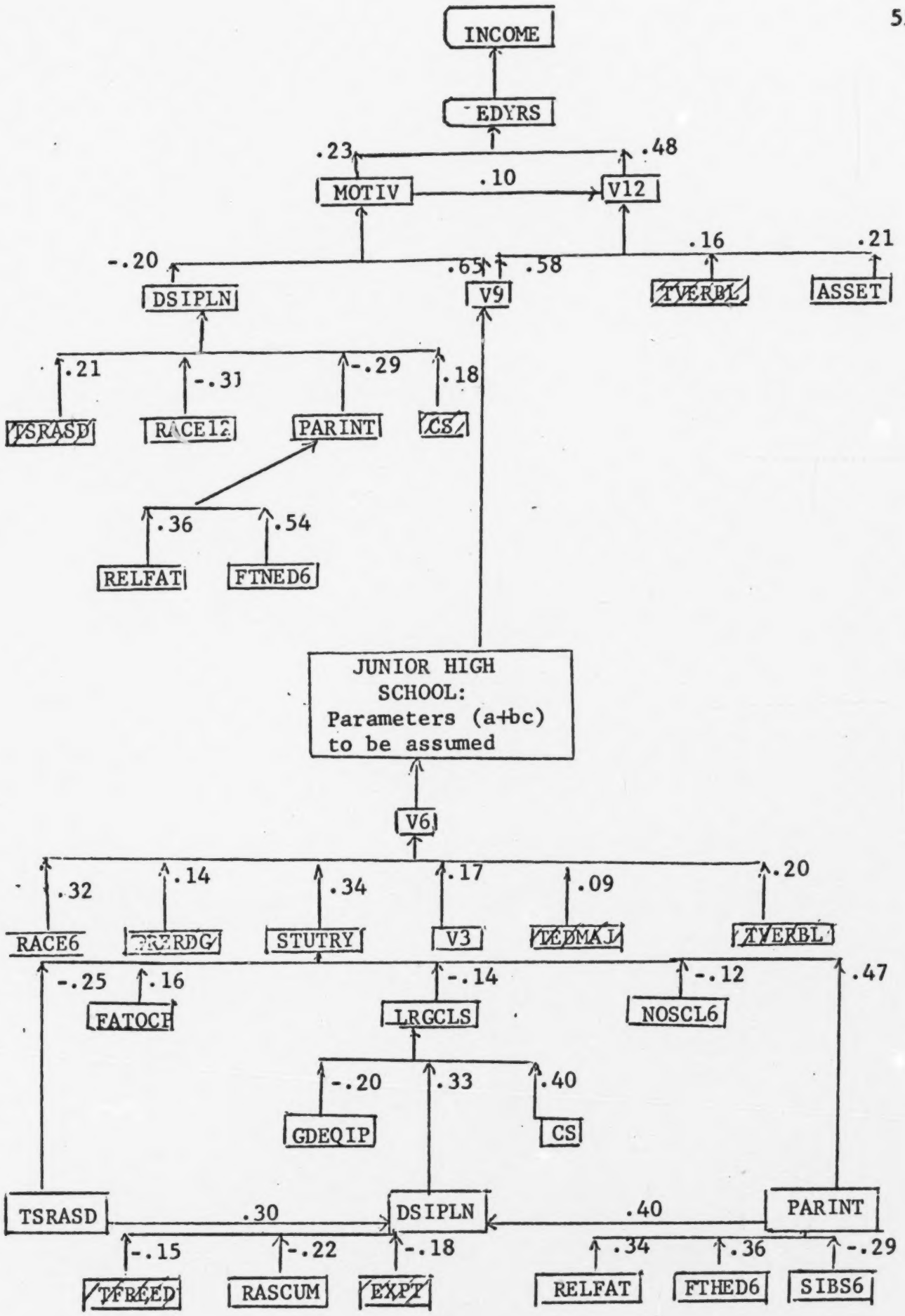

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FIGURE IX

CAUSATION IN THE COMPLETE EMPIRICAL MODEL

Table 1

Multipliers for Expected Years of Education

Policy Variable	Assumed Effects of V6 upon V9						Units
	.4	.5	.6	.7	.8	.9	
Average teacher verbal ability in high school (TVERBL-H)	.077	.077	.077	.077	.077	.077	The standard deviation (2.85) of answers correct on verbal tests given to teachers
Average teacher verbal ability in grammar school (TVERBL-G)	.037	.046	.056	.065	.074	.084	The standard deviation (2.2) of answers correct on verbal test given to teachers
Extent to which sixth grade students were read to before kindergarten (PRERDG-G)	.026	.032	.039	.046	.052	.059	The standard deviation (.23) of weighted average of student responses: 0=NEVER, 3=OFTEN
Teacher-student racial difference in grammar school (TSRASL-G)	.017	.021	.025	.029	.033	.037	The standard deviation (24) of the difference in the percentage of white teachers and students
Percentage of grammar school teachers who are education majors (T-DMAJ-G)	.015	.021	.025	.029	.033	.037	The standard deviation (18) of the percentage of grammar school teachers who are education majors
Teacher-student racial difference in high school (TSRASD-H)	.012	.012	.012	.012	.012	.012	The standard deviation (18) of the difference in the percentages of white teachers and students.
High school class size (CS-H)	.010	.010	.010	.010	.010	.010	Standard deviation (7.8) of students per class in high school
Grammar school class size (CS-G)	.004	.005	.006	.007	.008	.009	Standard deviation (6.2) of students per class in grammar school
Quality of instructional equipment in grammar school (GDEQIP-G)	.002	.002	.003	.003	.004	.004	The standard deviation (22) of the percentage of teachers who think their instructional equipment is at least adequate
Teacher experience in grammar school (EXPT-G)	.001	.001	.001	.001	.001	.001	The standard deviation (4.8) of the average years of experience of teachers in grammar school
Relative teacher freedom in grammar schools (TFREED-G)			.001	.001	.001	.001	The standard deviation (18) of the percentage of grammar school teachers who feel that they are free to alter instructional techniques to meet the need of students

years of education. Naturally, the multipliers of the grammar school variables increase in value as the estimate of a + bc increases while the multipliers of the high school variables remain constant. This leads to a difference in ranking in only one case, however. The multiplier for TVERBL-G corresponding to a + bc = .9 is .084. This is higher than the multiplier of TVERBL-H, .077.

We believe that an estimate of the sum of the direct and indirect effect of V6 upon V9 will yield values of about .6 for (a + bc). The average increment in years of education completed will never be: .08 x .8 years per student for a 2.85 increase in score on a high school teacher verbal aptitude test; .06 x .8 for a 2.2 increase in score on a grammar school teacher verbal aptitude test; .04 x .8 for a one category increase, affecting 25% of the students, in the extent to which they were read to before entering school; .025 x .8 for a 25 percentage point decrease in the difference between the percentage of black teachers and black students in grammar schools; .025 x .8 for an 18 percentage point increase in the percentage of grammar school teachers who are education majors; .012 x .8 for an 18 percentage point decrease in the difference between the percentage black teachers and black students in high school; .01 x .8 for an average decrease of 7.8 students per class in high school; .006 x .8 for an average decrease of 6.2 students per class in grammar school; .003 x .8 for a 2.2 percentage point increase in the percentage of teachers who consider their instructional equipment to be at least adequate; .001 x .8 for a 4.8 year increase in teacher experience in grammar schools; .001 x .8 for an 18 percentage point increase in the percentage of teachers who feel that they have adequate freedom to adjust instructional techniques to fit the needs and abilities of the students.

III. 3.3 Earnings Multipliers

The expected years of education-instrument variable relationships of the previous section are not easy to use as guides to educational policy formulation without a clear indication of the social benefit resulting from an extra year of education. In the present section we will attempt to provide a crude estimate of this link.

A proxy for the social benefits to be gained from a particular educational policy is the future income stream which would be generated by increasing the levels of the pertinent policy variables. The per-pupil social benefit (evaluated in dollars) can then be compared with the dollar cost of the policy, in order to see where educational dollars can be spent most effectively. Two possible rules can be used to guage the potential effectiveness of marginal adjustments in the direction of particular educational strategies.

1) maximize the benefit cost ratio (this criterion is appropriate only if the benefit-cost ratio is independent of the level of use of particular policy variables).

2) equate the benefit-cost ratio in all potential uses; with this approach, the multipliers presented may be presumed to represent the effectiveness of marginal variations in the neighborhood of the mean of the instrument variable.

Tables II through V contain a set of multipliers that indicate the increase in lifetime earnings that can be expected from a unit increase in a policy variable. To facilitate comparison with costs, the policy variables in these tables are presented in natural units rather than in standard deviation units. We have computed four tables rather than one because the additional earnings attributable to an extra year of education vary with the number of years of education. Thus, the increment to lifetime earnings from an extra two years of education is $37,000 if the student receives eight rather than ten years of schooling; $57,000 if the student receives twelve rather than ten years of schooling, $53,000 if the student receives fourteen rather than 12 years, and $114,000 if the student receives sixteen rather than fourteen years of schooling.[10]

The meaning of these tables will be illustrated by a consideration of the multipliers appearing in the second row of Table II and referring to the variable TVERBL - GRAMMAR SCHOOL. If we assume that a + bc is .4, the average increment to future student earnings per student arising from an average increase in teacher verbal ability of one test score point is $253. If we assume a + bc is .5, the corresponding increase would be $314. Table II contains the multipliers computed for 8 to 10 years of schooling; Table III contains the multipliers for 10 to 12 years of schooling; Table IV contains the multipliers for 12 to 14 years of schooling and Table V contains the multipliers for 14 to 16 years of schooling.

The interpretation of the multipliers will be illustrated by an examination of column 3 in Table II. There an average increase of one point on the teacher verbal score test in high schools (TVERBL-HIGH SCHOOL) leads to an increase of $407 per student in lifetime earnings. A similar change in grammar schools leads to

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[10]The source for these numbers is the U.S. Bureau of Education, American Education, July-August 1968. These numbers are additions to the simple sums of lifetime earnings; i.e., the present value of the extra lifetime increase if the interest rate is zero.

an increase of $383 in lifetime earnings per student. A one category increase in the extent to which 10% of the students are read to before school (PRERD[6]-scale: 0 = never, 2 = sometimes, 3 = often) increases expected future earnings by $256. A decrease of one percent in the difference between the percentages of black teachers and of black students in grammar schools leads to increases of $15 in student lifetime income. In high schools, the corresponding number is $10. An increase of one percent in the percentage of education majors teaching in grammar schools leads to an average increase of $21 in future earnings. An average of one fewer student per class in high schools leads to an increase in lifetime earnings of $19 per student. A corresponding change for the grammar schools would yield an increase of $14. An increase of one percentage point in the percentage of teachers who think their instructional equipment is adequate leads to an increase of $2 in average future earnings per student. An increase of one year in the average number of years of teacher experience in grammar school leads to an increase of $3.00 in average future earnings

If we assume that the increment to future earnings is given in the form of a constant yearly sum over a working period of some 40 years and that the rate of discount is always 5 percent[11] we can approximate the present value of the increment in the average student's future income stream. We will convert the values in the third column of Table IV (this corresponds to a+bc= .6) to present value equivalents in order to illustrate how these tables can be used to compare future benefits with present costs. This table is chosen because it contains information on the students closest to the sample mean, 12.8 years, in expected years of education. Because changes in the policy variables used in the present study affect whole classes rather than individual students we will multiply the average increase in the present value of future earnings by 30, the approximate sample average class size. This will give the present value of the total future benefits to be gained from particular policy measures. We will refer to the increment in the present value of total future benefits attributable to a unit increment in a policy variable as the return to that policy variable.

Then the return to teacher verbal ability[12] on the high school level is $7503 per teacher year, and the return to teacher verbal

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[11]For discount rates of 6%, 7% and 8% the returns given in the next paragraph should be multiplied by .88, .77, and .69 respectively.

[12]The definitions of the units in which the variables are measured are given in the previous discussion of Table II.

ability on the grammar school level is $7066 per teacher year. The return to preschool reading programs is $1710. The return to the teacher-student racial difference is $180 for the high school and $283 for the grammar school. The return to teacher education majors is $386. The returns to class size in high schools and grammar schools are $360 and $270 respectively. The return to instructional equipment is $39 and the return to teacher experience is $64. Finally, the return to teacher freedom is $13.

TABLE II

INCREASE IN EXPECTED LIFETIME EARNINGS PER STUDENT ASSOCIATED WITH A UNIT CHANGE IN THE POLICY VARIABLE FOR AN AVERAGE STUDENT WHO COMPLETES 10 YEARS OF SCHOOLING

Policy Variable	Multiplier of Policy Variable in Dollars of Lifetime Earnings per Student if the Assumed Effect of V6 Upon V9 is:						Units
	.4	.5	.6	.7	.8	.9	
Average teacher verbal ability in high school (TVERBL-H)	407	407	407	407	407	407	Average number of answers correct on verbal test given to teachers
Average teacher verbal ability in grammar school (TVERBL-G)	253	314	383	444	506	568	Average number of answers correct on verbal test given to teachers
Extent to which sixth grade students were read to before kindergarten (PRERDG-G)	57	70	85	100	114	129	Weighted average of student responses: 0=NEVER, 3=OFTEN
Teacher-student racial difference in grammar school (TSRASD-G)	10	12	15	17	20	22	Absolute value of the difference in the percentages of white teachers and students
Percentages of grammar school teachers who are education majors (TEDMAJ-G)	14	17	20	24	27	31	Percentage of grammar school teachers who are education majors
Teacher-student racial difference in high school (TSRASD-H)	10	10	10	10	10	10	Absolute value of the difference in the percentage of white teachers and students
High school class size (CS-H)	19	19	19	19	19	19	Average number of students per class in high school
Grammar school class size (CS-G)	10	12	14	16	19	21	Average number of students per class in grammar school
Quality of instructional equipment in grammar school (GDEQIP-G)	13	13	20	20	27	27	Percentage of teachers who think their instructional equipment is at least adequate
Teacher experience in grammar school (EXPT-G)	3	3	3	3	3	3	Average years of experience of teachers in grammar school
Relative teacher freedom in grammar schools (TFREED-G)	-	-	1	1	1	1	Percentage of grammar school teachers who feel that they are free to alter instructional techniques to meet the needs of students

TABLE III

INCREASE IN EXPECTED LIFETIME EARNINGS PER STUDENT ASSOCIATED WITH A UNIT CHANGE IN THE POLICY VARIABLE FOR AN AVERAGE STUDENT WHO COMPLETES 12 YEARS OF SCHOOLING

Policy Variable	Multiplier of Policy Variable in Dollars of Lifetime Earnings Per Student if the Assumed Effect of V6 Upon V9 is: .4	.5	.6	.7	.8	.9	Units
Average teacher verbal ability in high school (TVERBL-H)	627	627	627	627	627	627	Average number of answers correct on verbal test given to teachers
Average teacher verbal ability in grammar school (TVERBL-G)	389	485	590	685	780	875	Average number of answers correct on verbal test given to teachers
Extent to which sixth grade students were read to before kindergarten (PRERDG-G)	71	108	131	155	175	199	Weighted average of student responses: 0=NEVER, 3=OFTEN
Teacher-student racial difference in grammar school (TSRASD-G)	15	19	23	27	30	34	Absolute value of the difference in the percentages of white teachers and students
Percentage of grammar school teachers who are education majors (TEDMAJ-G)	21	27	32	37	42	47	Percentage of grammar school teachers who are education majors
Teacher-student racial difference in high school (TSRASD-H)	15	15	15	15	15	15	Absolute value of the difference in the percentages of white teachers and students
High school class size (CS-H)	29	29	29	29	29	29	Average number of students per class in high school
Grammar school class size (CS-G)	14	18	22	26	29	33	Average number of students per class in grammar school
Quality of instructional equipment in grammar school (GDEQIP-G)	2	2	3	3	4	4	Percentage of teachers who think their instructional equipment is at least adequate
Teacher experience in grammar school (EXPT-G)	5	5	5	5	5	5	Average years of experience of teachers in grammar school
Relative teacher freedom in grammar schools (TFREED-G)	-	-	1	1	1	1	Percentage of grammar school teachers who feel that they are free to alter instructional techniques to meet the needs of students

TABLE IV

INCREASE IN EXPECTED LIFETIME EARNINGS PER STUDENT ASSOCIATED WITH A UNIT CHANGE IN THE POLICY VARIABLE FOR AN AVERAGE STUDENT WHO COMPLETES 14 YEARS OF SCHOOLING

Policy Variable	Multiplier of Policy Variable in Dollars of Lifetime Earnings per student if the Assumed Effect of V6 Upon V9 is:						Units
	.4	.5	.6	.7	.8	.9	
Average teacher verbal ability in high school (TVERBL-H)	583	583	583	583	583	583	Average number of answers correct on verbal test given to teachers
Average teacher verbal ability in grammar school (TVERBL-G)	362	451	549	637	725	813	Average number of answers correct on verbal test given to teachers
Extent to which sixth grade students were read to before kindergarten (PRERDG-G)	81	100	122	144	163	185	Weighted average of student responses: 0=NEVER, 3=OFTEN
Teacher-student racial difference in grammar school (TSRASD-G)	14	18	22	25	28	32	Absolute value of the difference in the percentages of white teachers and students
Percentage of grammar school teachers who are education majors (TEDMAJ-G)	20	25	30	34	39	44	Percentage of grammar school teachers who are education majors
Teacher-student racial difference in high school (TSRASD-H)	14	14	14	14	14	14	Absolute value of the difference in the percentages of white teachers and students
High school class size (CS-H)	27	27	28	27	27	27	Average number of students per class in high school
Grammar school class size (CS-G)	13	17	21	24	27	31	Average number of students per class in grammar school
Quality of instructional equipment in grammar school (GDEQIP-G)	2	2	3	3	3	3	Percentage of teachers who think their instructional equipment is at least adequate
Teacher experience in grammar school (EXPT-G)	5	5	5	5	5	5	Average years of experience of teachers in grammar school
Relative teacher freedom in grammar schools (TFREED-G)	-	-	1	1	1	1	Percentage of grammar school teachers who feel that they are free to alter instructional techniques to meet the needs of students

TABLE V

INCREASE IN EXPECTED LIFETIME EARNINGS PER STUDENT ASSOCIATED WITH A UNIT CHANGE IN THE POLICY VARIABLE FOR AN AVERAGE STUDENT WHO COMPLETES 16 YEARS OF SCHOOLING

Policy Variable	Multiplier of Policy Variable in Dollars of Lifetime Earnings per Student if the Assumed Effect of V6 Upon V9 is: .4	.5	.6	.7	.8	.9	Units
Average teacher verbal ability in high school (TVERBL-H)	1255	1255	1255	1255	1255	1255	Average number of answers correct on verbal test given to teachers
Average teacher verbal ability in grammar school (TVERBL-G)	780	970	1810	1371	1561	1751	Average number of answers correct on verbal test given to teachers
Extent to which sixth grade students were read to before kindergarten (PRERDG-G)	175	216	263	310	350	398	Weighted average of student responses: 0=NEVER, 3=OFTEN
Teacher-student racial difference in grammar school (TSRASD-G)	31	39	46	54	61	69	Absolute value of the difference in the percentages of white teachers and students
Percentage of grammar school teachers who are education majors (TEDMAJ-G)	67	82	100	118	134	152	Percentage of grammar school teachers who are education majors
Teacher-student racial difference in high school (TSRASD-H)	30	30	30	30	30	30	Absolute value of the difference in the percentages of white teachers and students
High school class size (CS-H)	60	60	60	60	60	60	Average number of students per class in high school
Grammar school class size (CS-G)	30	37	45	52	60	67	Average number of students per class in grammar school
Quality of instructional equipment in grammar school (GDEQIP-G)	4	4	6	6	6	6	Percentage of teachers who think their instructional equipment is at least adequate
Teacher experience in grammar school (EXPT-G)	10	10	10	10	10	10	Average years of experience of teachers in grammar school
Relative teacher freedom in grammar schools (TFREED-G)	-	-	2	2	2	2	Percentage of grammar school teachers who feel that they are free to alter instructional techniques to meet the needs of students

APPENDIX B

A FACTOR ANALYSIS OF URBAN EDUCATION IN THE UNITED STATES

I. INTRODUCTION

One of the major concerns of the economics of the public sector is the efficiency of resource allocation in education. Since personal wealth in the form of acquired skills now constitutes the major form of wealth in advanced countries, personal mobility and status is now acquired by access to education and occupations. Educational policy is, therefore, at the forefront of social confrontation among socio-economic and ethnic groups in the United States.

In view of these considerations, it is of increasing importance to be able to disentangle the effects on "value-added" in education of school related inputs (teachers, equipment) from those of socio-economic and background variables.

The present study uses factor analysis to identify the regularities underlying the operation of the educational process in urban schools in the United States. The results of the analysis are used to suggest hypotheses about the nature of interactions affecting educational production functions.

The study is based on data from the Coleman Report on Equality of Educational Opportunity. The sample used consists of responses to questionnaires by principals, teachers and students in some 400 urban schools throughout the United States. The original Coleman data was refined for the present investigation in several respects: (1) the sample was made more representative of urban schools and of black schools; (2) numerous check and cross-checks were applied to the data to identify and correct recording and reporting errors; and (3) the responses to some 400 questions were used to construct information on a smaller set of variables of potential relevance to the educational process.

II. THE FACTOR ANALYSIS: RESULTS AND INTERPRETATION

The results of the factor analysis are summarized in the matrix of common factor coefficients presented in Table I. Each entry a_{ij} of the matrix shows the importance of factor j upon variable i. More specifically, the entries or "factor loadings" indicate the net correlation between each factor and the observed variables. Each $(a_{ij})^2$ represents the proportion of the total unit variance of variable i which is explained by factor j, after allowing for the contributions of the other factors. If the first row of the table is examined, it can be seen that 36 percent of interschool variations in average score on 6th grade verbal test are explained by Factor I, an additional 42

percent by Factor II, and another 5.8 percent by Factor V; the net contribution of Factors III and IV is negligible.[1]

The right-hand column of the table gives the sum of the squared factor loadings, or the "communality" of each variable. The communality indicates the proportion of the total unit variance explained by all the common factors taken together and is thus analogous to R^2 in regression analysis. The communality of the average score on the 6th grade verbal test, for example, is:

$$.587^2 + .661^2 + .047^2 + .062^2 + .236^2 = .842$$

That is to say, 84 percent of interschool variations in 6th grade verbal score are associated with five common factors which are extracted from the 55 social and economic variables incorporated in our analysis.

The matrix of factor loadings, in addition to indicating the weight of each factor in explaining the observed variables, provides the basis for grouping the variables into common factors. Each variable may reasonably be assigned to that factor in which it has the highest loading. Once variables are assigned to common factors, the factors need to be "identified" by giving a reasonable explanation of the underlying forces which they may be interpreted to represent. We shall, therefore, proceed to identify the factors which are specified in the results of our statistical analysis.

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[1]Slight differences between these values and the squares of the entries in Table I are due to rounding.

Table I

MATRIX OF FACTOR LOADINGS

	Variable	Factor 1	2	3	4	5	Communality
1	Average score on 6th grade verbal test	.587	.661	.047	.062	.236	.842
2	Average score on 6th grade math test	.530	.735	.048	.033	.233	.878
3	Average aspiration levels of 6th graders	.742	.207	.099	.172	.215	.679
4	Average 6th grader's parents' years of education and status of occupation	.794	.277	.115	.027	.051	.724
5	Extent of consumer durable ownership in 6th graders homes	.530	.663	.194	.088	.222	.815
6	Average size of 6th graders' households	-.525	-.563	-.053	.034	-.130	.613
7	Proportion of teachers who feel students' home life is not good	.617	.492	.067	-.028	.353	.753

	Variable	Factor 1	2	3	4	5	Communality
47	Extent of students' rudeness	.095	.128	-.064	.031	.607	.399
48	Extent of theft by students	.067	.114	-.031	-.034	.527	.298
49	Extent of students' violence against teachers	-.106	.011	.060	-.022	.449	.217
50	Extent of racial and ethnic conflict	.184	.143	-.111	-.009	.504	.320
51	Extent of time spent on classroom discipline	.380	.376	-.196	.099	.609	.705
52	Extent of administrative leadership	.155	.066	-.064	-.097	.432	.229
53	Average teacher's years of experience	.026	.032	.048	-.134	.538	.312
54	Proportion of teachers who expect to teach full-time until retirement	-.079	-.396	-.076	-.036	.553	.477
55	Amount of teacher population turnover	.182	.122	-.134	.062	.657	.501

	Variable	Factor 1	2	3	4	5	Communality
8	Average teacher's relative preference for students who are children of professional rather than laboring class	.473	.220	.099	-.051	-.122	.299
9	Amount of student turnover	.515	.232	-,072	-.065	.486	.565
10	Average number of schools attended by 6th graders	-.400	-.042	.026	.244	-.343	.340
11	Proportion of parental membership in PTA	.430	.084	-.041	.033	.177	.226
12	Amount of parental interference with school	-.630	-.068	-.044	.051	.182	.439
13	Amount of parental pressure for academic success put on student	-.674	-.171	.124	.042	-.057	.504
14	Average extent of parent-child discussion of schoolwork (6th graders)	.496	-.253	.028	.010	.092	.320

	Variable	1	2	3	4	5	Communality
15	Average amount parents had read to 6th graders in the pre-school years	.604	-.015	-.062	.079	.073	.381
16	Average number of 6th graders who attended a nursery school	.700	-.209	.070	-.139	-.216	.604
17	Average number of hours 6th graders watch television	-.568	.032	.068	-.019	-.099	.339
18	Proportion of students whose parents buy a newspaper every day	.496	.469	.038	.058	.211	.515
19	Average teacher's estimate of student academic effort	.638	.396	-.066	-.063	.471	.794
20	Relative magnitude of average student attendance	.475	.472	.055	-.145	.391	.625
21	Average score on 3rd grade verbal test	.482	.492	.053	.111	.306	.584

	Variable	Factor 1	2	3	4	5	Communality
22	Average score on 3rd grade non-verbal test	.416	.634	.144	.015	.198	.636
23	Proportion of white students (6th grade)	.188	.897	-.007	-.033	.254	.905
24	Proportion of white teachers	.023	.908	.201	-.041	-.053	.870
25	Proportion of white classmates desired by 6th graders	.132	.900	-.049	.004	.269	.894
26	Proportion of white students desired by teachers	.084	.885	-.054	.043	.144	.816
27	Proportion of teachers who encourage Negroes' occupational aspirations	.089	-.704	.189	-.002	-.106	.550
28	Proportion of 6th graders who don't live closer to another grammer school	.015	.463	-.047	-.039	.218	.265

	Variable	Factor 1	2	3	4	5	Communality
29	Average occupational status level of teachers' fathers	.171	.554	.148	-.163	-.213	.429
30	Proportion of 6th graders whose mothers don't work	-.011	.669	.242	-.066	.112	.523
31	Proportion of 6th graders whose real father doesn't live at home	.371	.724	.064	.046	.229	.720
32	Average score on teacher's verbal test	.057	.651	.423	-.100	-.009	.615
33	Number of averaga teacher's semesters of college education	-.053	-.399	.263	-.053	.047	.236
34	Proportion of 6th graders who attended kindergarten	.243	.286	.583	-.001	-.085	.488
35	Extent of gymnasium facilities	-.018	.173	.591	.068	-.054	.387
36	Extent of special teachers' services	-.127	.041	.569	.302	.115	.446

	Variable	Factor 1	2	3	4	5	Communality
37	Principal's salary	-.066	-.040	.721	.143	-.119	.560
38	Average teacher's salary	.014	.067	.883	-.039	.080	.793
39	Amount of time average teacher spends teaching in class	-.079	-.104	-.419	.359	.334	.433
40	Length of students' school day	-.089	-.085	-.413	.351	.217	.355
41	Average extent of teacher absence	.027	-.133	.469	-.001	-.212	.284
42	Number of classrooms per student	-.075	-.025	-.035	-.763	.163	.616
43	Number of remedial reading teachers per student	-.005	-.025	-.143	-.679	.064	.486
44	Number of counselors per student	.023	.096	-.172	-.752	.007	.604
45	Computed average class size	.041	-.030	-.101	.512	-.204	.317
46	Extent of vandalism	.051	.181	-.060	-.026	.425	.220

The First Factor

Description. The first factor describes the influence of the socioeconomic status of the students' parents upon students and schools. The variables with high loadings in this factor fall into four groups: (1) Those that indicate the socioeconomic status of students' parents; (2) those that measure the extent of parents' interaction with school and students; (3) measures of the students' scholastic achievements and aspirations in the sixth grade; and (4) measures of the students' abilities, using third grade achievement tests.

Parents whose children attend schools that have high scores on factor one place high on the education and occupation scales (variable 4 has a loading of .794), own relatively many consumer durables (variable 5,.530), and have a relatively small number of people living in their homes (variable 6, - .525). The parents of students in the high scoring schools tend to put more pressure on the school personnel and students, as shown by their greater participation in the PTA (variable 11, .430), their greater interference with the school (variable 12, - .630), and their tendency to emphasize to their children the value of good grades (variable 13, - .674). These parents are more likely to buy a newspaper every day (variable 18, - .496). They have read more often to their children during the pre-school years (variable 15, .604), have sent more of their children to nursery school (variable 16, .700), and now talk to their children more frequently about schoolwork (variable 14, .496), while the children watch less television (variable 17, - .568).

The students in the high scoring schools respond to their parents' concern with education by trying harder in school (variable 19, .638) and by being absent less often (variable 20, .475). They do better on their third grade (variable 21, .482; variable 22, .416), and sixth grade achievements tests (variable 1, .587; variable 2, .530), and have higher academic and occupational aspirations (variable 3, .742) than the students attending low scoring schools.

The schools that scored high on factor one have a low rate of student population turnover (variable 9, .515; variable 10, - .400), and employ many teachers who feel that their students have a good home life (variable 7. .617), and who prefer to teach children from the professional rather than the laboring class (variable 8, .473).

In summary, schools with high scores on this socioeconomic factor (factor one) have bright, achievement-oriented students whose parents are in high socioeconomic strata and are very concerned about their children's education.

Discussion. The first factor, which describes the socioeconomic background of students but includes no characteristics of the school *per se*, accounts for more than twenty-seven percent ($= .530^2$) of the variance in the average school score on the sixth grade math test and

for more than thirty-three percent (= $.587^2$) of the variance in the average school scores on the sixth grade reading test. The associations in this factor therefore confirm the results of Coleman et al who found that social class accounts for much of the variance in student achievement and aspirations.[2]

The Second Factor

Description. The second factor describes the relationships among race, discrimination, socioeconomic status, and school output. The variables in this factor fall into four groups: (1) Racial composition and preference of students and teachers within a school and an index of the racial segregation of schools; (2) academic characteristics of teachers; (3) race-associated home-life characteristics of students; (4) scores on the third and sixth grade achievement tests and a measure of the regularity of student school attendance.

A typical student in a school scoring low on this factor is black (variable 23, .897), prefers to have a high proportion of black classmates (variable 25, .900), and lives closer to a school other than his own (variable 28, .463). His teacher is also black (variable 24, .908), prefers to teach a predominantly black class (variable 26, .885), and will encourage her students to aspire to good jobs (variable 27, -.704).

The typical teachers employed by schools scoring low on factor two will not have great verbal facility (variable 32, .651), although they generally have more semesters of education (variable 33, - .399) than the typical teacher in a high scoring school. Their fathers have lower status occupations (variable 29, .554) than those of teachers of high scoring schools.

A high proportion of the teachers in low scoring schools think their students have a poor home life (variable 7, .492). A high proportion of the fathers of students in low scoring schools do not live at home (variable 3, .724), and a high proportion of students have working mothers (variable 30, .669). The parents of students whose

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[2]Critics of the Coleman Report contend that this result was obtained because school facilities and special teacher services are highly correlated with the socioeconomic status of the neighborhood served by the school and because socioeconomic variables were arbitrarily forced to enter before school variables in a stepwise regression that explained student achievement. This chain of circumstances, the critics argue, would have led to an underestimate of the explanatory power of school variables in the sense that the addition of R^2 associated with the inclusion of school variables could be higher than that associated with socioeconomic variables if their order of entry were reversed. The factor analytic technique used in the present paper is such that all variables are introduced simultaneously. It is, therefore, not open to this criticism.

schools scored low on factor two tend to have few consumer durables (variable 5, .663), and many people in their household (variable 6, -.563). They do not generally buy a newspaper every day (variable 18, .469).

Finally, schools with low scores on this factor have a high proportion of low-achieving (variable 1, .661; variable 2, .735) students who do not attend school regularly (variable 20, .472).

Discussion. The loadings in this factor indicate that the academic achievement level of many students may be related to their home environment and that Negro children may have special difficulties in this area due to the direct and indirect effects of the racial discrimination to which they are subjected.

Many of the influences which combine in this factor to explain lower scholastic achievement in predominantly black schools are race related differences in home life. More fathers are absent from home, more mothers work, there is more overcrowding, and the main breadwinner has a lower status job.

Unlike the first (socioeconomic) factor, school variables (average teacher verbal ability and semesters of education) do appear in the second factor. However, the school measures which enter into this factor are also related to race since white teachers tend to have more verbal facility and fewer semesters of education than the black teachers.

The Third Factor

Description. The third factor describes the school in terms of the ability or willingness of school authorities to provide good personnel and facilities for its students. The variables in this factor fall into two groups: measures of school facilities and services, and measures of wages and hours of work for school personnel.

Schools with high scores on the third factor have good facilities: a kindergarten (variable 34, .583); good gymnasium facilities (variable 35, .591); and special teachers for music, art and speech (variable 36, .569). They also make an effort to attract good personnel, as indicated by the fact that they tend to pay higher salaries to principles (variable 37, .721) and teachers (variable 38, .883), have fewer hours of required classroom work (variables 39, - .419);

and 40, - .413), and a relatively higher number of school days not worked by teachers (variable 41, .469).[3]

Discussion. It is striking that the third factor, which describes the traditional approaches of school districts to improving the quality of education, has so low a net effect. It explains less than .1% of variance of student achievement (variables 1 and 2) and student aspirations (variable 3). In addition there seems to be a relatively weak association between measures of teacher quality such as experience (variable 53, less than .25%), teacher attachment to the teaching profession (variable 54, less than .7%), teacher turnover (variable 55, less than 2% of variance), and working conditions in the school district. Only 16% of the variance in teacher verbal ability is explained by this factor. The loadings in factor three tend to suggest that the policies summarized by this factor are relatively ineffective in improving the educational performance of schools. One reason for this may be that teacher salary schedules typically vary only between, and not within, school districts. Since most teachers are women and are usually the secondary wage earners in a family, their husbands' places of work must be taken into account when making any locational decision. Therefore, the effect of shorter hours and higher wages upon teacher quality might be greater if they varied within school districts.

The Fourth Factor

Description. The fourth factor describes the school in terms of the magnitude of per pupil facilities. Schools with high scores on this factor have a high number of rooms per student (variable 42, - .763), many remedial reading teachers (variable 43, - .679), and counselors (variable 44, - .752) per student, and small class sizes (variable 45, .512).

Discussion. None of the student achievement variables (variables 1 - 3) correlate well with these school inputs. The net correlation is less than one percent of variance. This implies that raising the per student level of teacher services represented in this factor, at least

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[3]The number of days of teacher absence per year is probably a proxy for the number of sick-days granted to teachers. Note that in this factor, the number of hours that students spend in class is positively related to the number of hours that teachers spend in class. This may be indicative of an awareness by school authorities that trade-offs exist among teacher quality, class size, and the average number of hours that students spend in class. That is, as average yearly wages are raised and hours are lowered to attract better teachers, fewer teachers can be hired, given a budget limit. In that case, unless the number of hours that students spend in class can be lowered, class size will rise.

within the range represented in the data, is not an effective means of raising student performance. This finding could have important implications for the allocation of school budgets.

The Fifth Factor

Description. The fifth factor describes the school in terms of the problems teachers face and the effect of these problems upon the retention of the teaching staff.

Schools with low scores on this factor are troubled by racial conflict (variable 50, .504), vandalism (variable 46, .425), and by problems of theft (variable 48, .527), and violence against teachers (variable 49, .499). The students in these schools are often impertinent to their teachers (variable 47, .607), and a great deal of classroom time is devoted to discipline (variable 51, .609). Administrators cannot provide effective leadership (variable 52, .432), and experienced teachers tend to leave these schools as soon as possible (variables 54, .553; and 55, .657).

Discussion. Factor five seems to contain variables that teachers take into consideration when choosing among schools within a school district. This interpretation is suggested by the association in this factor of greater teacher experience (and hence greater seniority) with low-student-problem schools. Because school systems usually allow teachers with the highest amount of seniority to have first pick of schools when vacancies occur, we would expect schools with the most desirable characteristics to be chosen by experienced teachers. This effect may be reinforced by the greater ability of experienced teachers to cope with student behavior problems.

While the third factor contains variables (wages and hours) that tend to be used to attract talented teachers with high qualifications to alternative school systems, the variables in the fifth factor take on values that attract or repel teachers with high seniority within a given school system, in which wage scales and hours worked tend to be uniform. Neither the third nor the fifth factor's forces seem to affect student academic performance directly.

SUMMARY

The factor analysis performed on the fifty-five variables has produced five factors which summarize some of the important interrelationships in the process of education. Taken together these factors account for more than eighty-four percent of the variance in the average school scores on a reading test given to eighth grade students, more than eighty-seven percent of the variance in a comparable mathematics test, and almost sixty-eight percent of the variance in a measure of student aspirations.

The first factor, which accounts for more than thirty-nine percent of the sum of the variances of the fifty-five (normalized) variables, classifies each school according to the socioeconomic status of its students' families. This factor alone accounts for more than thirty-four percent of the variance in the average sixth grade reading scores and for almost fifty-five percent of the variance in student aspirations.

The second factor accounts for thirteen percent of total variance. It classifies each school according to the racial composition of its student and faculty. It is interesting that this factor accounts for almost fifty-four percent of the variance in average mathematics scores but only four percent of the variance in student aspirations.

The third factor accounts for over ten percent of total variance. It characterizes each school according to the willingness and ability of school officials to provide good facilities and teachers for students. As mentioned above, the variables used to attract teachers in this factor are mainly those that operate between school systems.

The fourth factor accounts for over six percent of total variance. It characterizes each school by the extent of per-pupil facilities in the school.

The fifth factor also accounts for approximately six percent of total variance. It characterizes each school by extent of disciplinary problems and school atmosphere for teachers. The variables included in this factor tend to be associated with intra-school system preference of teachers.

Factor Rows

The preceding sections of this chapter characterized the separate factors by the variables of which they are composed. This section will further clarify some of the meaningful associations among some of these important variables by an examination of the factor rows.

Variable 1. Sixth Grade Reading Test. The average scores on the sixth grade reading test have their highest loadings in factors one and two (35% and 44% of variance, respectively). This indicates that sixth grade reading scores are associated primarily with socioeconomic status (factor one) and race (factor two), although the verbal ability of teachers (which has a loading of .651 in factor two) appears to compensate somewhat for the lack of verbal interaction in students' homes. The reading test variable has virtually no association with the third and fourth factors (school adaptability and per pupil facilities, respectively) but is weakly associated (6% variance) with the fifth factor (intra-system teaching preferences and school problems). This

may mean that experienced teachers can influence reading scores to some extent or merely that students who score well on reading tests will tend to be the more serious students who behave well.

Variable 3. Aspirations of Sixth Grade Students. As pointed out in the summary of factors, the measure of student aspirations has its highest loading in the first (socioeconomic) factor. Thus, most of the variance (54%) in this variable seems to be due to the home environment of the student, and more specifically, to the socioeconomic status of his parents. The association between socioeconomic status and aspiration may be related to the narrow range of experience and the lack of successful role models so characteristic of low status students.

Variables 19 and 20. Variables that Measure Student Motivation. The variables in this group are the average teacher's estimates of how hard her students try, and whether or not there is a high rate of absenteeism among students in her school. These variables have moderate loadings in the first (variable 19, .638; variable 20, .475), second (.396; .472), and fifth (.794; .625) factors.

Their inclusion in the first (socioeconomic) and second (race) factors reflect tendencies due primarily to home influences. Students from low status homes which lack the parental interest in education, and black students, particularly males, who tend to lack a successful father to emulate, are unmotivated. The loadings of the motivation variables in the fifth factor (school problems) indicate that, as might be expected, the more highly motivated students tend to cause fewer problems in school.

Variables 21 and 22. Student Ability. The variables under this heading are the average school scores on a verbal test (variable 21), and a nonverbal test (variable 22) given to third grade students. These variables are included in the study because they presumably reflect less of the school influence and more of the innate ability and home influence than the sixth grade achievement scores. Both third grade tests were included in the first (23% and 24% of variance, respectively), and second (17% and 40% of variance) factors. The verbal test has a higher, though small, loading (.306) in the fifth (school problems) factor than the non-verbal test (.198).

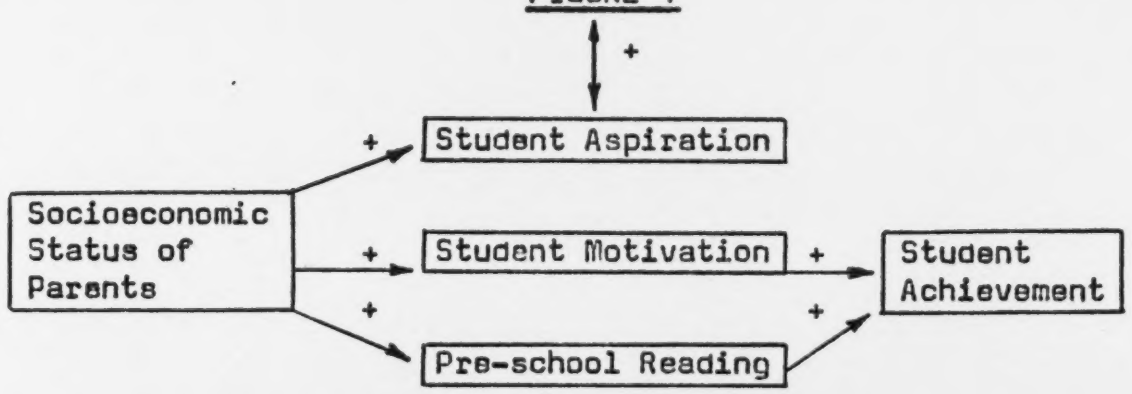
The difference in the amount of variance explained by factor five (verbal test, 9%; non-verbal, 4%) could be an early indication of the effects of student motivation. Non-verbal tests are constructed in such a way that they tend to weight native ability higher than school-related phenomena. Therefore, the higher loading for the verbal test in the school problems (fifth) factor may be due to an early association between achievement in school and status-and-race determined motivational patterns.

III. HYPOTHESES

The factor-analytical interpretations given in the previous section are complex and any descriptive model that tried to take all of them into account could have to be very ambitious. However, certain broad outlines in the process of education do seem evident and in this section a tentative causal structure is suggested. This structure is consistent with, but not uniquely given by, the preceding factor analysis.

Several causal relationships are suggested by the first (socioeconomic factor. The arrows in the following diagram show the postulated directions of causation:

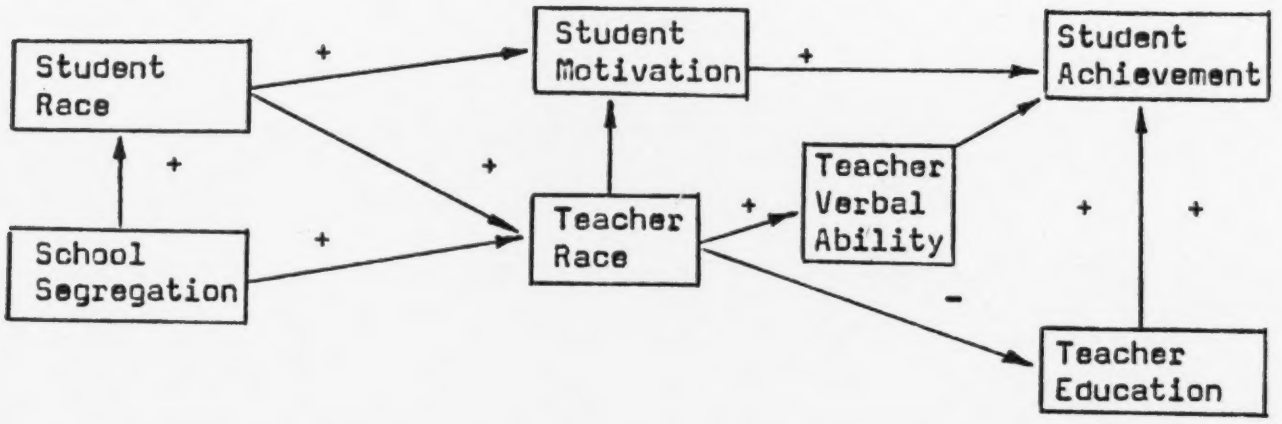
FIGURE 1



The relationship between parental status and student aspiration and achievement evident in this factor may be caused by several related conditions: (1) High status parents tend to motivate the student directly by stressing the importance of academic achievement; (2) high status parents provide parental models which delineate a wider apparent range of opportunities open to students, thereby enhancing student aspirations; and (3) high status parents tend to provide a home environment that better prepares the student for school by such things as pre-school reading to the student. Student aspirations and motivation interact in a mutual positive feedback relationship which together with pre-school preparation determines student achievement.

In the second factor, the interrelationships between student achievement and other school and background variables stem from racial differences among schools. The postulated causal relationships underlying the associations among the variables in the second factor are shown in the following diagram:

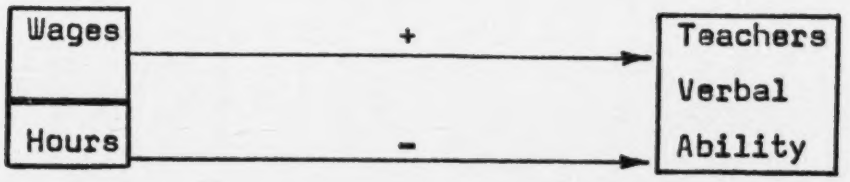
FIGURE 2



School segregation (evident in variable 28) has both a direct and an indirect effect upon average student achievement. The direct influence stems from the effect of the structure of the black family (variables 30 and 31) upon student motivation (variables 19 and 20). The indirect influences stem from peer group interactions, which result in an indirect impact of the racial composition of the classroom upon individual student achievements, and from the consequences of a racial matching of teachers and students (variables 23 and 24).[4] Black teachers are better able to motivate black students than white teachers, and even though they tend to have lower verbal scores than their white counterparts,[5] this results in higher student achievement.

The relationships in the third factor can be shown diagramatically as follows:

FIGURE 3



The loadings in this factor suggest that higher salaries and a lower number of in-class hours can be used to attract teachers with high verbal ability. However, the ceteris paribus nature of the

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[4]The racial matching is part voluntary (variable 26).

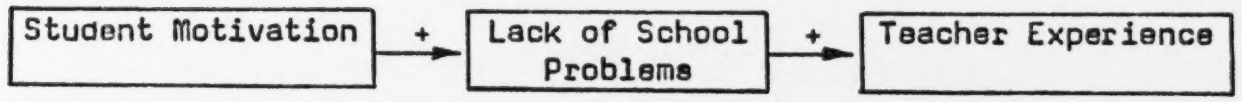
[5]The lower verbal scores of black teachers are compensated in part by a larger number of semesters of education.

within-factor relations implies that it is entirely possible that if school A is a high-problem school and school B is a low-problem school, school B may attract better teachers even if it offers lower wages.

Factor four contains only variables which measure per pupil facilities and seems to reflect the effects of scale in the school's operation. These appear to be essentially unrelated to student performance.

The fifth factor contains variables that relate to the attractiveness of the school to teachers. The moderate loadings in this factor of the variables that measure student motivation suggest the following structure:

FIGURE 4



Experienced teachers with seniority prefer to work in schools with fewer discipline problems. By the same token, however, the more experienced teachers are also more adept at handling discipline problems and at enhancing (directly and indirectly) student motivation.

The causal structures suggested by an examination of the orthogonal factors can now be combined in a single structure to describe the interrelationships occurring in the complete factor analysis.

(See Figure 5.)

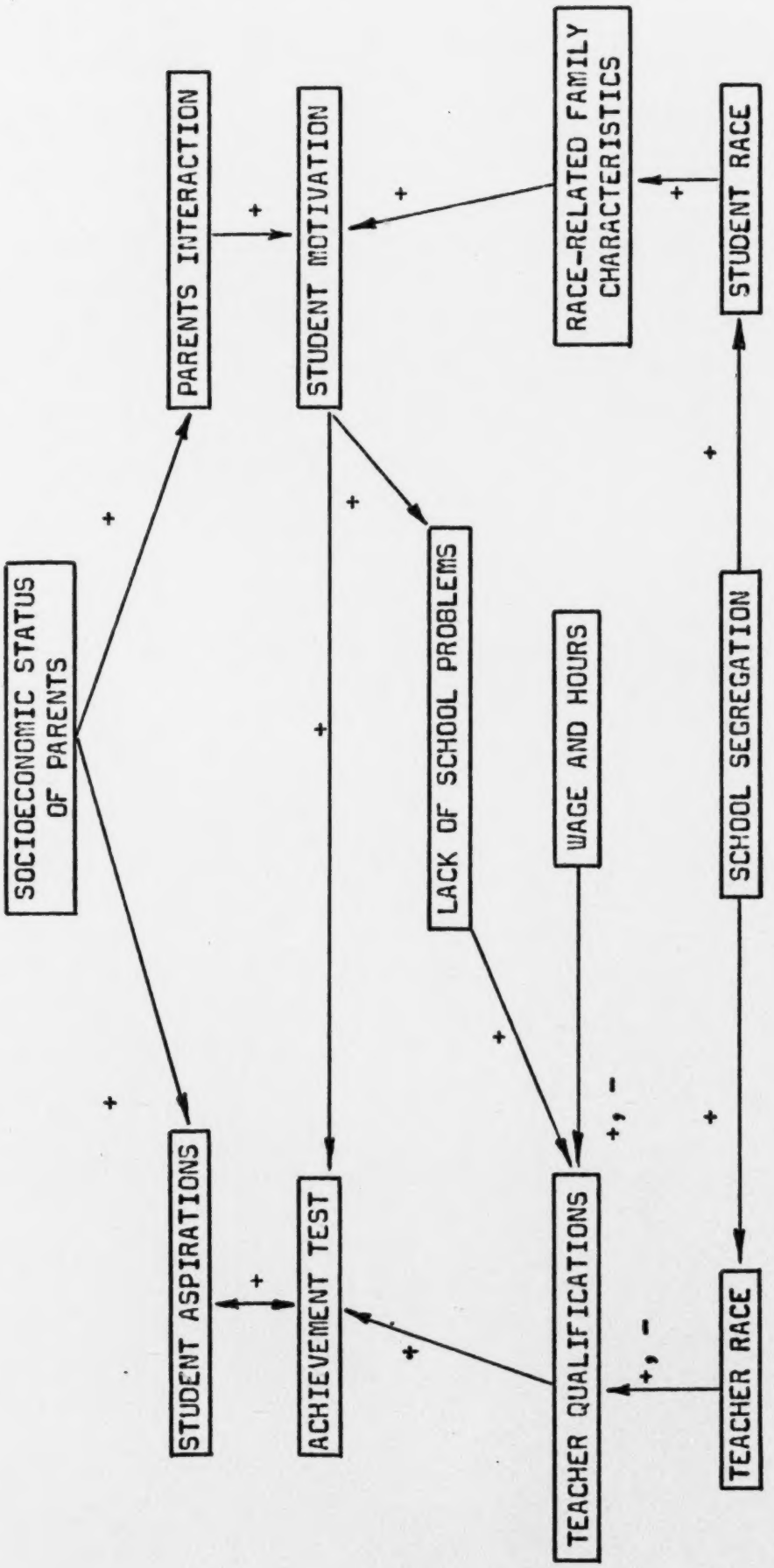

SOCIOECONOMIC STATUS OF PARENTS
PARENTS INTERACTION
STUDENT MOTIVATION
RACE-RELATED FAMILY CHARACTERISTICS
STUDENT RACE
STUDENT ASPIRATIONS
ACHIEVEMENT TEST
LACK OF SCHOOL PROBLEMS
WAGE AND HOURS
SCHOOL SEGREGATION
TEACHER QUALIFICATIONS
TEACHER RACE
+
+, -

FIGURE 5

From the point of view of policy, the interesting groups in Figure 5 are those that can be changed by school authorities. The groups are: (1) teachers' wages and hours of required work; (2) teachers' qualifications and assignment practices; (3) racial segregation in the school; and (4) the average socioeconomic status of students' parents. The following example, which makes use of diagram 5, will suggest how these variables might be used.

Suppose that a desirable policy is one which will increase the score of the average black student on an achievement test. One conceivable policy measure would be to decrease the extent of racial segregation. The effect of this would be to decrease the proportion of black students in the school, which would indirectly increase average student motivation, thereby decreasing student-associated problems in the school. By this means, the average level of qualifications of teachers willing to work in the school would be increased and achievement scores would increase.

A more direct policy measure would be to increase wages and/or lower hours of teachers in proportion to the percentage of black students in the school. This would increase the average level of teacher qualifications in the school which would in turn increase average student achievement scores.

Political pressures upon school boards frequently preclude manipulation of the proportion of black students in a school. However, if the variance in the socioeconomic status of black students in a school system is great enough, an alternative to increasing wages or lowering hours would be to alter the status mix in predominantly black schools. The effect of this would be to indirectly increase average student motivation. Thus, student associated problems can be expected to decrease and more highly qualified teachers will be willing to work in the school. This, as before, will tend to increase student scores on achievement tests.

CONCLUSION

This study identifies by means of factor analysis the underlying regularities in a set of fifty-five variables relevant to the education process. The regularities are used to suggest hypotheses from which a set of tentative policy measures can be derived.

The factor analytic interpretation of the educational process in the present study is strikingly consistent with the major findings of the Coleman Report even though free from some of its methodological deficiencies:

(1) Family background explains much of the variance in student achievement;

(2) School facilities account for a small proportion of the variance in student achievement;

(3) Teacher characteristics, particularly verbal ability, account for a greater proportion of the variance in student achievement than any other school factor.

A fourth important finding of the Coleman Report (that the social composition of the individual student's classmates explained a large proportion of the variance in individual student achievement) could not be directly corroborated by the present study, but is entirely consistent with it.